Structural-Acoustic Properties of Flexible Rectangular Boxes

Georgiev, V.B.[1], Krylov, V.V.[2]
[1] Research associate, Department of Aeronautical and Automotive Engineering, Loughborough University, Loughborough, Leicestershire, LE11 3TU, UK
[2] Professor, Department of Aeronautical and Automotive Engineering, Loughborough University, Loughborough, Leicestershire, LE11 3TU, UK

Abstract

Rectangular box-like structures are used widely in a large number of engineering applications, e.g. as elements of railway carriages, heavy goods vehicles, buildings, civil engineering constructions, etc. Although flexible rectangular boxes represent one of the geometrically simple types of engineering structures, their structural-acoustic properties can not be described by closed-form analytical solutions. In the present study, a comprehensive numerical investigation of typical all-flexible rectangular box structures has been carried out to elucidate the physics of structural–acoustic interaction in them and to explore the possibilities of reduction of the associated structure-borne interior noise. Finite element method has been used to compute the resonant frequencies, the mode shapes and the structural-acoustic frequency response functions of different rectangular box models. The obtained results could assist in better understanding of structural-acoustic properties of flexible rectangular boxes as well as of numerous more complex structures using rectangular boxes as their building elements.

Keywords: Flexible rectangular boxes; Structural-acoustic properties, Finite element analysis, Structural-acoustic modes, Frequency response functions.

Структурно-акустические свойства гибких прямоугольных конструкций

Георгиев В.Б.[1], Крылов В.В.[2]
[1] Научный сотрудник, Кафедра авиационной и автомобильной техники, Университет Лафборо, Лафборо, Лестершир, Великобритания
[2] Профессор, Кафедра авиационной и автомобильной техники, Университет Лафборо, Лафборо, Лестершир, LE11 3TU, Великобритания

Аннотация

Прямоугольные коробчатые конструкции широко применяются в большом количестве инженерных конструкций, например, в качестве элементов железнодорожных вагонов, большегрузных автомобилей, зданий, инженерных сооружений и т. д. Хотя гибкие прямоугольные коробки представляют собой геометрически простые типы инженерных сооружений, их структурно-акустические свойства не могут быть описаны в виде простых аналитических решений. В настоящей работе проводится комплексное численное исследование типичных гибких прямоугольных коробчатых конструкций для выяснения физики структурно–акустического взаимодействия в них и изучения возможности сокращения связанного корпусного шума внутри кабины. Метод конечных элементов был использован для вычисления резонансных частот, форм колебаний и функций структурно-акустической частотной характеристики различных моделей прямоугольных коробчатых конструкций. Полученные результаты могут способствовать лучшему пониманию структурно-акустических свойств гибких прямоугольных конструкций, таких как многочисленные комплексные структуры, использующие данные прямоугольные конструкции как строительные элементы.

Ключевые слова: *гибкие прямоугольные конструкции; структурно-акустические свойства, метод конечных элементов, структурно-акустические методы, функции частотного диапазона.*

E-mail: V.B.Georgiev@lboro.ac.uk, V.V.Krylov@lboro.ac.uk (corresponding author)

Introduction

Flexible rectangular box structures, often called box-like or box-type structures, are used widely in a large number of engineering applications, e.g. as elements of railway carriages, heavy goods vehicles, buildings, civil-engineering constructions, etc. Although all-flexible rectangular boxes represent one of the geometrically simple types of engineering structures, the analysis of their structural-acoustic properties can not be performed in terms of closed form solutions. Earlier, Dickinson and Warburton [1] have obtained approximate analytical expressions for the natural frequencies of uncoupled vibrations of such structures, considering them as the systems consisting of plates with the boundary conditions approximated by Fourier series. Equalizing the resulting expressions to the actual boundary conditions led to an infinite set of equations that had to be truncated to obtain the approximate solutions. These authors also have conducted experimental measurements of the natural frequencies. Later, Hooker and O'Brien [2] have calculated the lowest natural frequencies for a box of the same dimensions using finite element (FE) method and compared them with the approximate analytical and experimental results obtained by Dickinson and Warburton [1].

More recently, the authors of the paper [3] used FE calculations to carry out vibration analysis of a thin-plate box, considering only in-plane motion. They asserted that under certain conditions, the vibrational response is dominated by long wavelength in-plane waves. Later on they extended their study, analysing flexural vibrations of the same model using a combination of FE and analytical approaches [4]. The proposed method aimed to predict uncoupled vibrations of thin-plate structures in the medium frequency range. In the papers [5-8], vibrations of rectangular box-like structures have been investigated analytically using some simple approximations, e.g. taking into account only in-plane waves being transmitted to the adjacent walls under the impact of the initially flexural waves.

Some authors have utilized rectangular box models to verify different optimization procedures for noise reduction [9-12]. In this regard, the rectangular box models assisted in a quicker estimation of the proposed design modifications and of the efficiency of noise reduction.

In spite of the extensive use of the above-mentioned all-flexible rectangular box structures, their coupled structural-acoustic behaviour was not properly analysed. Indeed, the existing approximate analytical solutions for structural normal modes of vibration [1] are rather complex, and their use for a coupled structural-acoustic analysis is too problematic. As far as we are aware, numerical investigations of structural-acoustic properties of all-flexible rectangular boxes have not been reported either.

The aim of this paper is to carry out a comprehensive numerical investigation of structural-acoustic properties of all-flexible rectangular boxes. In the first part of the paper, the attention will be paid to understanding the uncoupled structural and acoustic properties of flexible rectangular boxes and to establishing relationships between its geometrical symmetry and modal patterns. This part will thus revisit the results obtained in the pioneering papers [1, 2]. A comparison will be made, where possible, of the results obtained in the present paper with the results of [1, 2]. In the second part of the paper, the coupled structural-acoustic properties of all-flexible rectangular boxes will be studied. In particular, their frequency response functions (FRF's) will be investigated and their features interpreted using the results of the first part. Some of the results described in the present paper have been reported in the authors' earlier paper [13].

1. Basic Model Description

The basic model under consideration represents a rectangular box with all-flexible walls of the same thickness made of steel (with the values of Young's modulus $E = 2\ 10^{11}$ N/m^2, Poisson's ratio $\sigma = 0.31$ and mass density $\rho = 7950\ kg/m^3$), see Fig. 1.

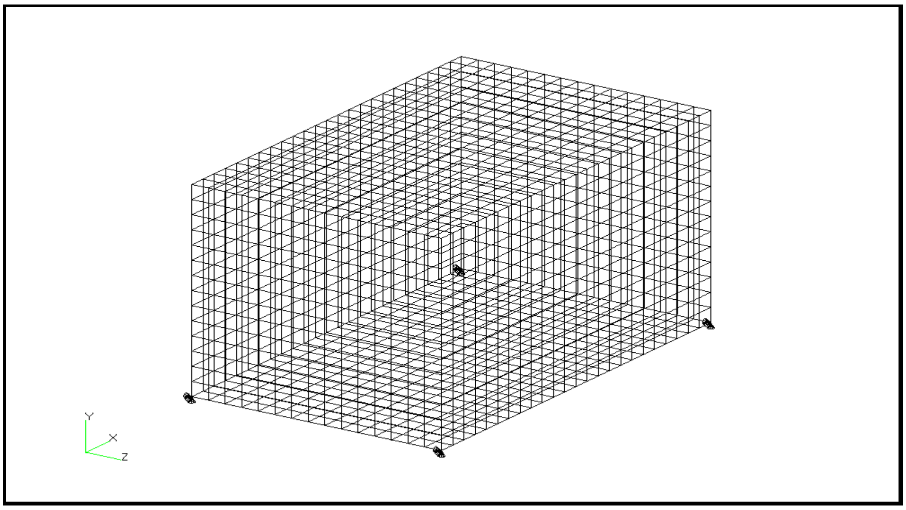

Fig. 1. Finite element model of a rectangular box structure

The only boundary conditions imposed on the model are applied at the corners of the bottom plate, which simulates fixing the box at four points to a rigid foundation. The model dimensions are as follows: x = 2.4, y = 1.4 and z = 1.5 meters. The wall thickness of the model was chosen to be 8 mm, which corresponds to a fundamental structural vibration frequency of about 15–20 Hz.

Initially, an analysis of the uncoupled structural and acoustic sub-systems is carried out, and then some structural-acoustic modes and a set of FRF's of the coupled structural-acoustic system is calculated and discussed.

2. Structural and Acoustic Analysis of the Uncoupled Model

In this section, using finite element software, MSC.Nastran and MSC.Patran, the uncoupled normal vibration modes and the corresponding natural frequencies of the all-flexible box structure under consideration is being analyzed. Note that for the purpose of the uncoupled analysis, a slightly different model is used. The difference from the above-mentioned basic model used in the fully coupled analysis is in the absence of boundary conditions at four corners at the bottom. The purpose of this was to generalize the uncoupled analysis of the rectangular box structure. Thus, "free-free' boundary conditions were adopted everywhere, whereas in the coupled analysis the model was considered as being placed on a certain foundation. In other words, the boundary conditions utilized in the coupled analysis simulated an attaching mechanism, which restricts the structural behavior of the model. In the uncoupled analysis, a refined finite element mesh was used consisting of 7248 CQUAD finite elements - for the structural sub-system, and 5040 CHEXA finite elements - for the acoustic sub-system.

2.1 Qualitative Interpretation of the Structural Behaviour of Rectangular Boxes

Figure 2 shows some structural vibration modes calculated for the uncoupled rectangular box model. As the box structure under consideration is fully symmetrical in respect of the three orthogonal coordinate planes, a number of symmetric and anti-symmetric structural modes should occur (see also [2]). In the 3-D picture (Fig. 2), the symmetric and anti-symmetric normal modes can not be seen clearly. This is why in Fig. 3 the same normal modes are presented in XY plane, where symmetric and anti-symmetric modes are clearly seen.

Except for the existence of symmetric and anti-symmetric modes, all-flexible rectangular box structures exhibit another interesting phenomenon known from the general symmetry considerations, namely the presence of repeated frequencies associated with degenerate modes. This phenomenon, of course, occurs in rectangular boxes of higher symmetry. To illustrate it numerically, some additional calculations have been conducted for a cubic box model with the dimensions (1, 1, 1) m (see Fig. 4) and for a rectangular box model with the dimensions (2.4, 1.5, 1.5) m (see Fig. 5).

It is interesting to attempt a kind of qualitative interpretation of structural vibrations of flexible rectangular boxes. In particular, looking at Fig. 4, one can suggest that, in respect of plate wave propagation, the box structure under consideration can be considered as an inhomogeneous plate-like structure with spatially varying geometry and stiffness.

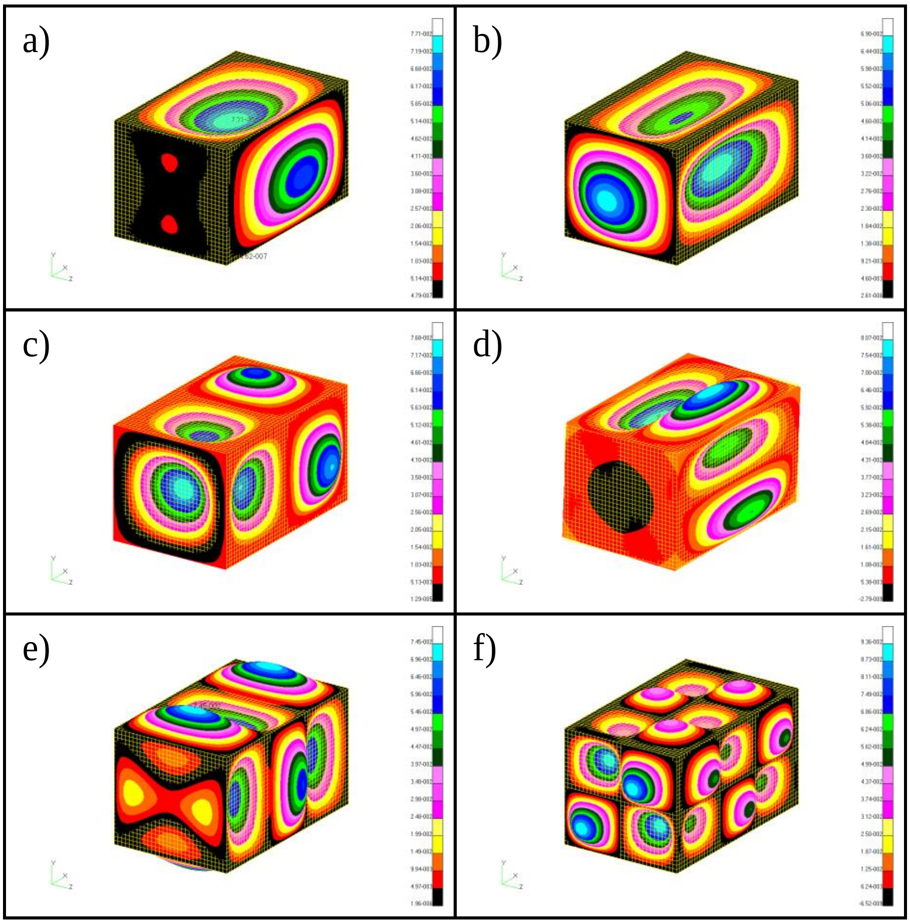

Fig. 2. Some structural modes of a rectangular box, at: a) 13.692 Hz, b) 22.069 Hz, c) 34.730 Hz, d) 42.138 Hz, e) 43.541 Hz and f) 90.904 Hz

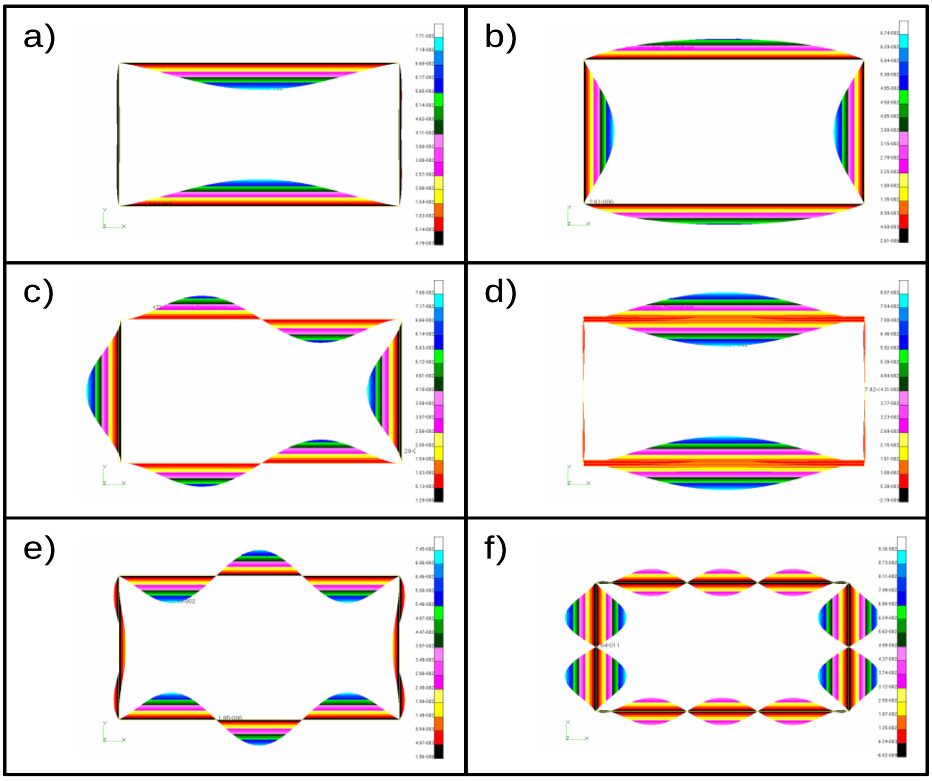

Fig. 3. Symmetric and anti-symmetric normal modes at: a) 13.692 Hz, b) 22.069 Hz, c) 34.730 Hz, d) 42.138 Hz, e) 43.541 Hz and f) 90.904 Hz

The structure's plates and edges are respectively more and less prone to vibrate. In this regard, the edges can be likened to a sort of stiffeners or ribs. It has been pointed out by Skudrzyk [14] that such ribs can transmit (without significant losses) the twisting moment around their longitudinal axis, and only at very high frequencies the rotary inertia can suppress the transmission. On the other hand, the bending moment around their transverse axes is not transmitted except for very high frequencies.

In other words, it follows from this qualitative interpretation that waves which nodal lines are parallel to the edges can be relatively easily transmitted to an adjacent plate, whereas those with nodal lines perpendicular to the edges are efficiently isolated. Thus, the higher stiffness of the edges defines different transmission properties of the waves in different directions, which directly influences the plate wave propagation.

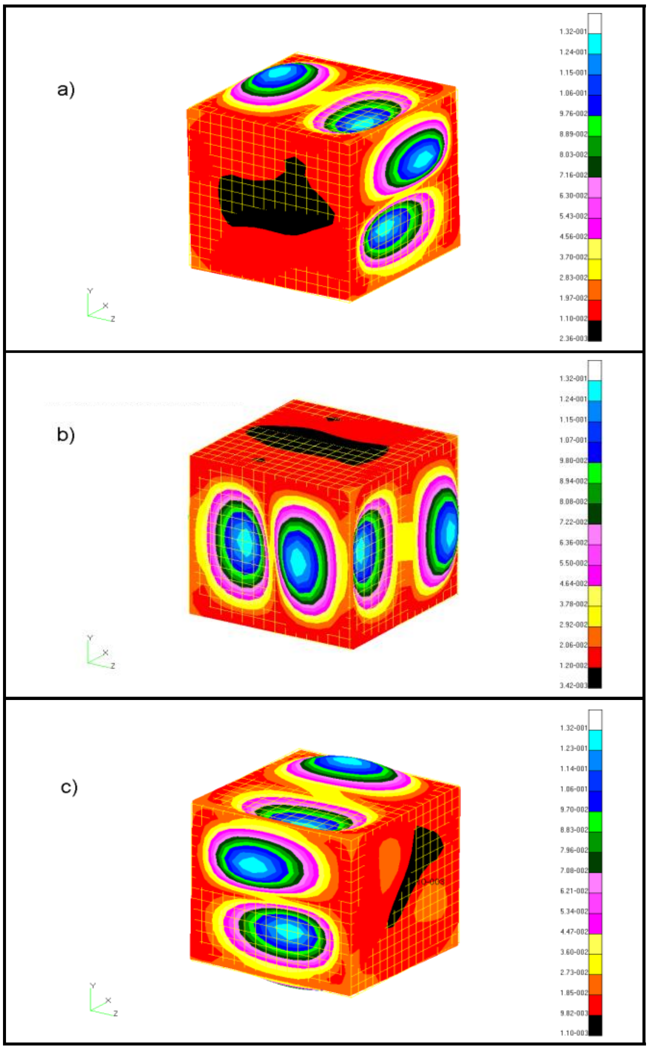

Fig. 4. ‘Waveguide modes’ of a cubic box structure at 105.11 Hz

Returning to the box structure under consideration and bearing in mind the above-mentioned qualitative properties of the edges, it can be assumed that the edges form three closed-loop waveguides on the box surfaces that govern plate wave propagation and form the normal modes of vibration. For example, in Fig. 4 the wave circulation around each of the coordinate axes can be clearly seen. Furthermore, the above-mentioned three quasi-circumferential waveguides could be assumed to form relatively independent closed-loop resonators, which means that predominant vibration modes occur in one of these guides and rarely the vibratory motion spreads on two or three waveguides simultaneously. In this way, one can conclude that predominant normal modes of box structures consist of the normal modes of each of the above-mentioned three closed-loop ‘waveguide resonators’.

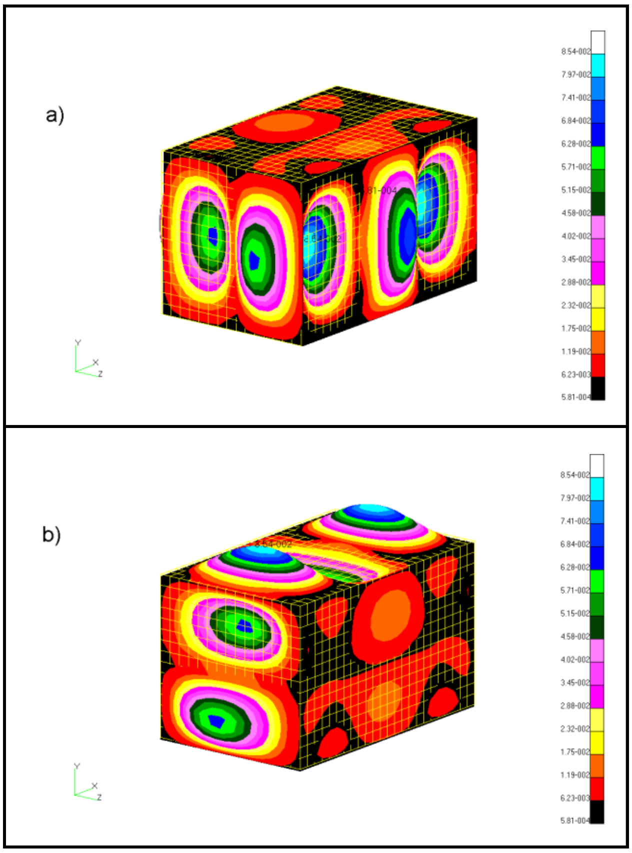

Fig. 5. 'Waveguide modes' of a modified box model 'A' at 42.36 Hz in the directions of: a) Y-axis and b) Z-axis

To demonstrate the influence of geometrical dimensions on the existence of repeated frequencies, it is convenient to compare the above-mentioned waveguide resonators between them. According to the above interpretation, the initial model of a box structure (see Figs. 1 and 2), with the dimensions 2.4, 1.4 and 1.5 m respectively along the coordinates x, y and z, forms three different coupled waveguide resonators and its natural frequencies are all different. In the light of the above, the next logical step is to consider the most symmetrical model with all waveguide resonators being identical. In this regard, a cubic box, which forms three absolutely identical waveguides along the different coordinates, satisfies the necessary requirements. As expected, in this case the predominant normal modes occur at a number of sets of three repeated natural frequencies. For example, in Fig. 4 one can see the modes corresponding to one of these sets of repeated resonance frequencies, i.e. at 105.11 Hz. For each of the modes shown in Fig. 4, each of the resonators vibrates in the same manner as the other. In other words, in the case of a cubic model there are three different normal modes corresponding to the same frequency, as expected (a triple degeneracy), and each of them belongs to one of the waveguide resonators.

Using a slightly modified rectangular box model (let us call it model 'A'), a change associated with the transition between the cubic model (Fig. 4) and the original rectangular model (see Figs. 1 and 2) was investigated (see Fig. 5). The dimensions of the model 'A' were 2.4, 1.5 and 1.5 m, which has made it more symmetric compared to the initial rectangular model but less symmetric in comparison with the cubic model. As expected, this structure has two identical modes in respect of y and z coordinates and one different mode in respect of x axis. Finite element calculations show a number of sets of two repeated natural frequencies associated with the two identical waveguide resonators, and most of the other resonances are associated with the third resonator. In Fig. 5 one can see modes associated with wave propagation in the two identical resonators around y and z directions. In this case, the behavior of the equal resonators is similar to those in the case of a cubic model. The difference is in the presence of the third resonator associated with a significant number of normal modes that never occur at a repeated frequency.

Note in this connection that vibrations of a simple non-circular (oval-like) cylindrical shell structure have been studied numerically and experimentally [15, 16] as well as analytically [17]. It has been demonstrated that the natural frequencies of this oval-like shell model can be approximated by the local resonant frequencies of each of its quasi-flat plate components. This approximation was possible as a result of weak coupling between the top and bottom quasi-flat plates of this model due to much higher effective stiffness of the adjacent curved plates (shells) separating the top and the bottom quasi-flat plates. In this regard, a rectangular box is very different, as each of its constitutive plates has four other plates adjacent to it, and all adjacent plates are strongly coupled to each other, so that no separate consideration of plate vibration is permitted.

A rectangular box structure can be considered as a compound of six rectangular plates which individual fundamental frequencies, assuming the same material properties for each of them, depend only on their geometrical characteristics. The basic box model studied in this paper has dimensions of 2.4, 1.4 and 1.5 m in respect of x, y and z coordinates, and its individual plate components are the following: plate component 1 (2.4, 1.5 m), plate component 2 (2.4, 1.4 m) and plate component 3 (1.5, 1.4 m). These dimensions determine very close sets of resonant frequencies of the individual component plates.

Table 1. Structural and acoustic natural frequencies of an uncoupled box model

№	**Box structure, natural frequencies, Hz**		**Plate component 1, natural frequencies, Hz**	**Plate component 2, natural frequencies, Hz**	**Plate component 3, natural frequencies, Hz**	**Acoustic, FE calculated, natural frequencies, Hz**	**Acoustic, exact, natural frequencies, Hz**
	1	**2**	**3**	**4**	**5**	**6**	**7**
1	13.692	13.693	11.805 (1, 1)	13.061 (1, 1)	18.209 (1, 1)	69.07 (1, 0, 0)	69.02 (1, 0, 0)
2	19.306	17.019	21.726 (2, 1)	22.974 (2, 1)	43.609 (2, 1)	110.6 (0, 0, 1)	110.4 (0, 0, 1)
3	20.749	18.506	37.282 (1, 2)	39.514 (3, 1)	47.387 (1, 2)	118.5 (0, 1, 0)	118.3 (0, 1, 0)
4	22.069	22.074	38.275 (3, 1)	42.307 (1, 2)	72.463 (2, 2)	130.4 (1, 0, 1)	130.2 (1, 0, 1)
5	24.821	23.834	47.090 (2, 2)	52.094 (2, 2)	85.958 (3, 1)	137.2 (1, 1, 0)	137.0 (1, 1, 0)
6	26.290	25.876	61.448 (4, 1)	62.675 (4, 1)	96.024 (1, 3)	138.4 (2, 0, 0)	138.0 (2, 0, 0)
7	28.509	26.652	63.458 (3, 2)	68.427 (3, 2)	114.33 (3, 2)	162.2 (0, 1, 1)	161.9 (0, 1, 1)
8	29.983	27.250	79.735 (1, 3)	91.038 (1, 3)	120.62 (2, 3)	176.2 (1, 1, 1)	175.9 (1, 1, 1)
9	34.730	28.773	86.394 (4, 2)	91.317 (4, 2)	145.23 (4, 1)	177.2 (2, 0, 1)	176.8 (2, 0, 1)
10	42.138	34.276	89.381 (2, 3)	92.450 (5, 1)	161.70 (3, 3)	182.2 (2, 1, 0)	181.8 (2, 1, 0)

In Table 1, columns 3, 4 and 5, the first ten analytically calculated natural frequencies of the separate plate components satisfying simply supported boundary conditions are presented. One can see that resonant frequencies of these plates are noticeably different from the FE results for the resonant frequencies of the full box structure (columns 1 and 2 in Table 1). This agrees with the above-mentioned statement about the lack of possibility to approximate rectangular box resonant frequencies by resonant frequencies of its separate plate components.

Note that the natural frequencies of the full box structure presented in Table 1 have been calculated for the two cases: with "free-free" boundary conditions (column 1) and with simply supported boundary conditions imposed on the all edges of the model (column 2). Despite some discrepancies between these sets of frequencies, their closeness, at least for the first eight modes, is indicative. In this frequency range, the structure under both sets of boundary conditions has high modal density. This is why under "free-free" boundary conditions there are 312 resonance peaks in this range (excluding the first six rigid-body natural frequencies) and the last one occurs at 498.90 Hz, whereas under simply supported boundary conditions the result is 311 peaks with the last natural frequency at 499.99 Hz. These calculations support the assumption made earlier that in the low and medium frequency ranges the edges of a rectangular box-structure transmit predominantly flexural waves which nodal lines are parallel to them.

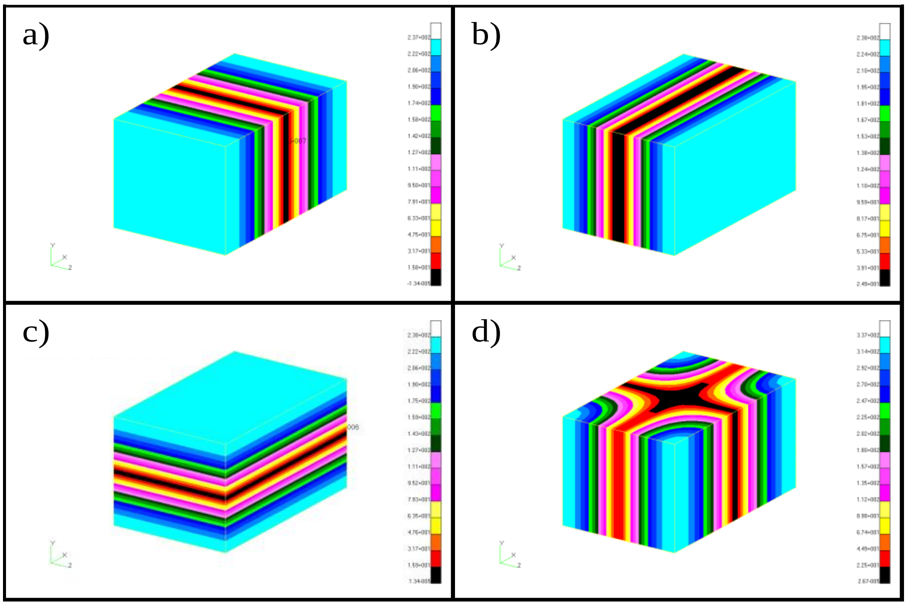

Fig. 6. First four uncoupled acoustic modes of a rectangular box enclosure, at: a) 69.07 Hz, b) 110.64 Hz, c) 118.57 Hz and d) 130.43 Hz

Figure 6 and Table 1 show some of the normal modes and natural frequencies of the uncoupled acoustic sub-system. The comparison between the analytically calculated natural frequencies (Table 1, column 7), which are determined very easily for the acoustic rectangular sub-system, and those calculated using finite element techniques (Table 1, column 6) shows a good agreement between them and thus validates the chosen mesh size. Even in the medium frequency range (for mode (6, 2, 1)) the exact solution defines the natural frequency of 489.44 Hz, whereas the finite element code gives 499.67 Hz. In other words, using this finite element mesh, a maximum relative error of 2 % in the highest frequency range of interest was achieved, which guarantees correct and reliable numerical results.

2.2 Comparison with Other Theoretical and Experimental Results

It is interesting to compare the results of the present numerical approach with the results obtained experimentally and theoretically by the earlier authors. For that purpose, a box structure with the dimensions x = 0.36576, y = 0.3048 and z = 0.24384, m has been calculated, i.e. the same one that has been used by Dickinson and Warburton [1] and by Hooker and O'Brien [2] who investigated it from the viewpoint of purely structural vibration behavior.

Figure 7 demonstrates the first 8 normal modes calculated in the present paper for Dickinson and Warburton's model, whereas Fig. 8 shows the same normal modes in XY plane only - in order to demonstrate symmetric and anti-symmetric spatial patterns more clearly. The similarity between these normal modes and those shown in Fig 2 and Fig. 3 is quite obvious.

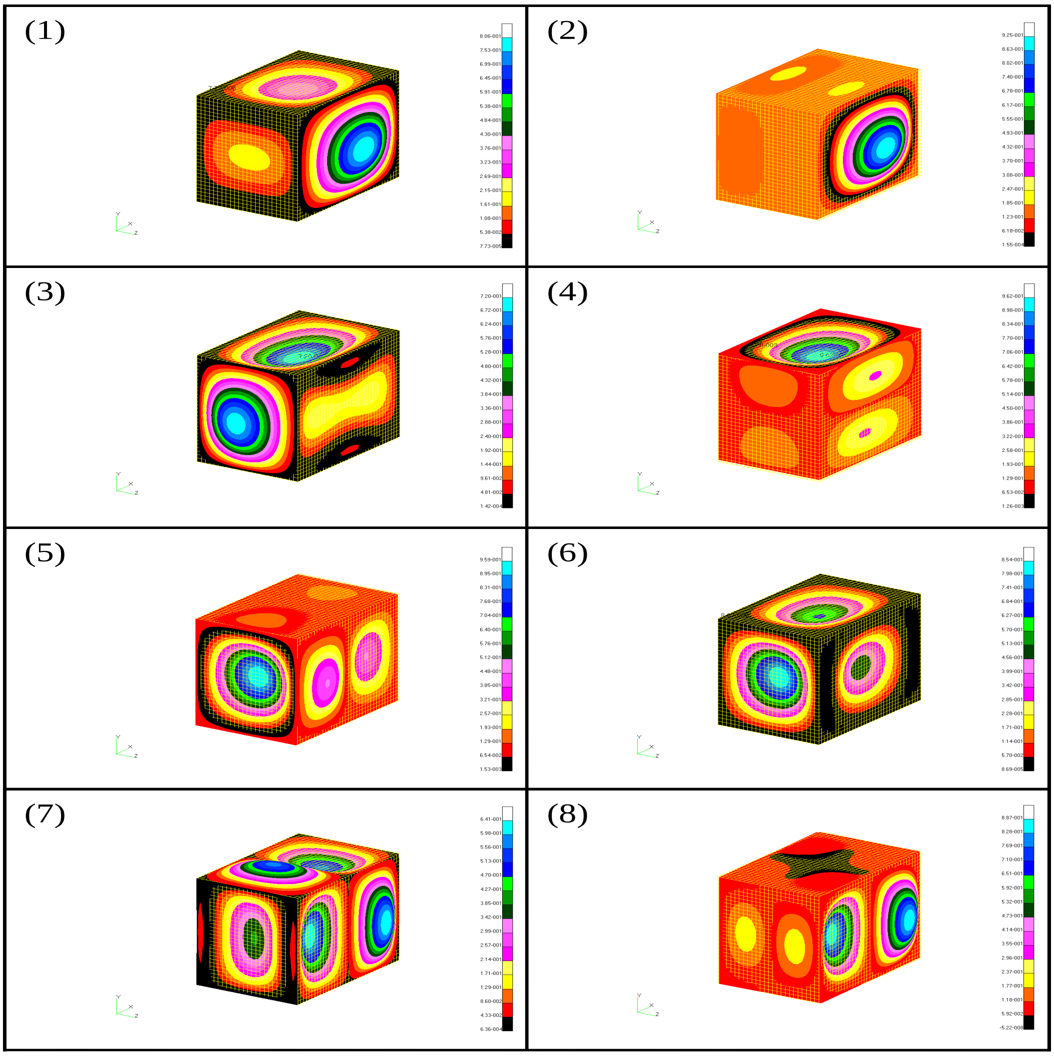

Fig. 7. First eight normal modes calculated for Dickinson and Warburton's model

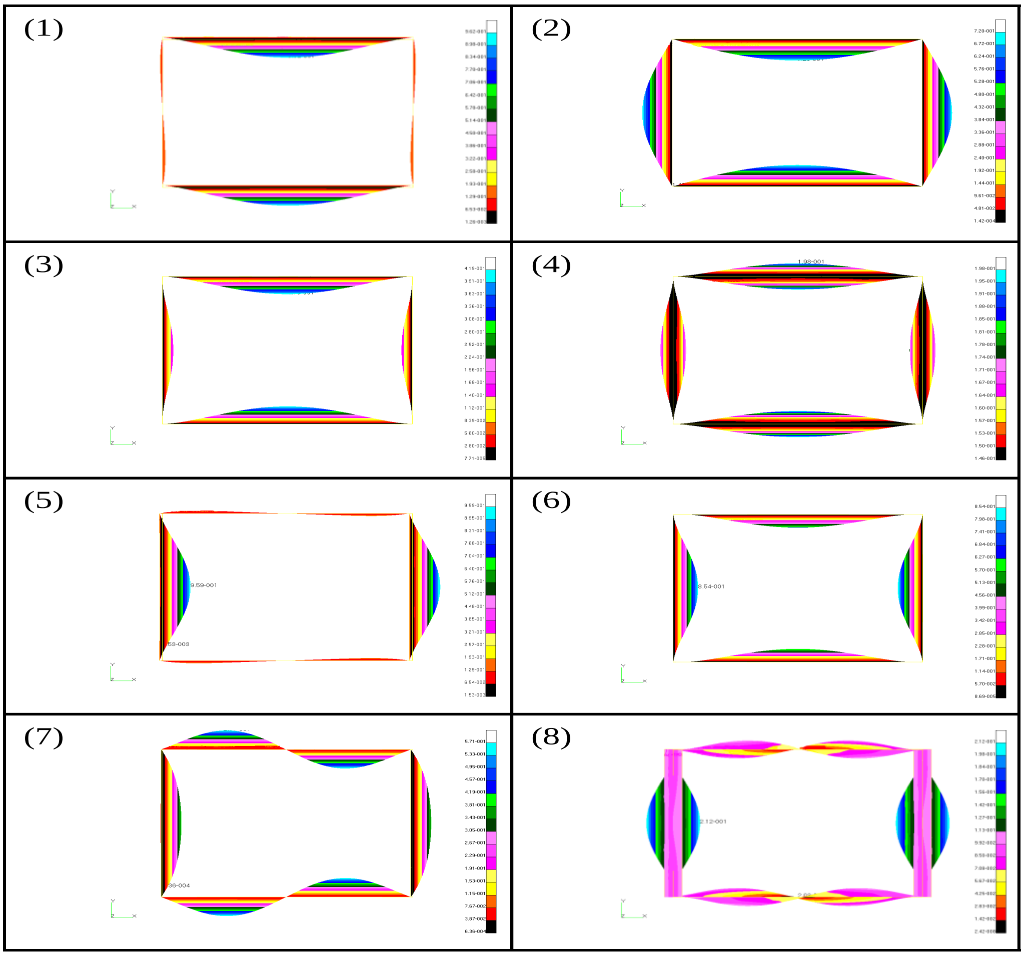

Fig. 8. First eight normal modes calculated for Dickinson and Warburton's model in XY plane

In Table 2, the natural frequencies of the model under consideration obtained by different authors are presented. The approximate analytical results of Dickinson and Warburton [1].are shown in column 2, whereas column 3 presents their experimental results. In column 4, the numerical results obtained using the procedure adopted for all numerical calculations in the present paper are shown. In column 5, another set of numerical data obtained by Hooker and O'Brien [2] can be seen.

As one can see, there is a good agreement between the experimental measurements (column 3) and the numerical results of the present paper (column 4). On the other hand, comparing the FE results of the present work and of the work of Hooker and O'Brien [2] with the experimental results, one can see a noticeable improvement in accuracy of numerically calculated natural frequencies in the present paper as compared to those calculated by Hooker and O'Brien [2], which could be expected for a modern finite element software. Comparing the present FE results with the approximate analytical calculations of Dickinson and Warburton [1], one can see that the precision of the latter is generally not as good as that of the present work, but it is better than the precision achieved by Hooker and O'Brien [2].

Table 2. Measured and calculated natural frequencies of vibration for Dickinson and Warburton's box model

№	Theoretical frequencies, Hz, (Dickinson and Warburton 1967)	Experimental frequencies, Hz, (Dickinson and Warburton 1967)	FE frequencies, Hz, (Present work)	FE frequencies, Hz, (Hooker and O'Brien 1974)
1	**2**	**3**	**4**	**5**
1	179	178	178.53	184
2	203	228	230.36	206
3	258	264	270.88	262
4	272	282	281.87	279
5	283	297	301.85	291
6	333	328	331.54	336
7	384	395	397.82	394
8	397	399	399.16	409
9	437	451	449.87	452
10	455	479	473.91	465
11	486	495	485.43	497
12	499	497	499.29	512
13	570	571	565.08	588
14	577	580	575.10	595
15	624	634	625.15	669
16	648	642	640.51	671

3. Structural-Acoustic Analysis of the Fully Coupled Model

In this section, fully coupled structural-acoustic modes are investigated, and a set of structural-acoustic frequency response functions (FRF's) at specific acoustic nodes are discussed and compared. As was mentioned above, simply supported boundary conditions at the corner nodes of the bottom plate (Fig. 1) were imposed to simulate an attaching mechanism. In the coupled model, 1812 CQUAD structural finite elements and 5040 CHEXA acoustic finite elements were used. Energy losses in the structure were modeled using 3 % damping factor. As far as air acoustic losses are concerned, a simple damping coefficient of 1 % was used for the sake of simplicity.

In Figure 9, some of the normal modes of the fully coupled model, that are influenced by the first and second uncoupled acoustic modes, are presented. As it is well known [18], the coupling depends on the spatial similarity and frequency closeness between the uncoupled structural and acoustic normal modes. Therefore, some of the structural modes can couple better with certain acoustic modes, in contrast to others. The three normal modes shown in Fig. 9, at about 68, 71 and 111 Hz, are not much affected by the coupling effects and are very similar to the corresponding uncoupled modes. Note that these particular modes also do not make significant contributions to the overall structural-acoustic frequency response functions (see Figs. 10 – 13).

The structural-acoustic pressure FRF's calculated in the centre of the box interior (at node 4826) and away from the centre (at node 6825) are shown in Figs. 10 and 11 respectively. For each of these figures, the driving force with the amplitude of 200 N is applied in the centre and in the vicinity of a corner of the bottom plate. In Figs. 12 and 13 respectively, the same FRF's are plotted together for different nodes and for the driving force applied either in the centre or in the vicinity of a corner of the bottom plate.

Taking into account the value of the first uncoupled acoustic natural frequency of the model, which is about 69 Hz, the graphs presented in Figs. 10 – 13 can be regarded as consisting of two parts. The first part, bellow 69 Hz, represents the area where FRF's are induced by structural vibrations of the model. The second part, above 69 Hz, is the area where FRF's are formed by a complex interaction of the structural and fluid vibrations. In the first part of the graphs, one can notice that resonant amplitudes depend only on the position of the external force and do not depend on the position of a receiver. This can be clearly seen in Figs. 10 and 11, where the difference between them is around 15 dB, and in Figs. 12 and 13, where FRF's at both receiver positions simply coincide.

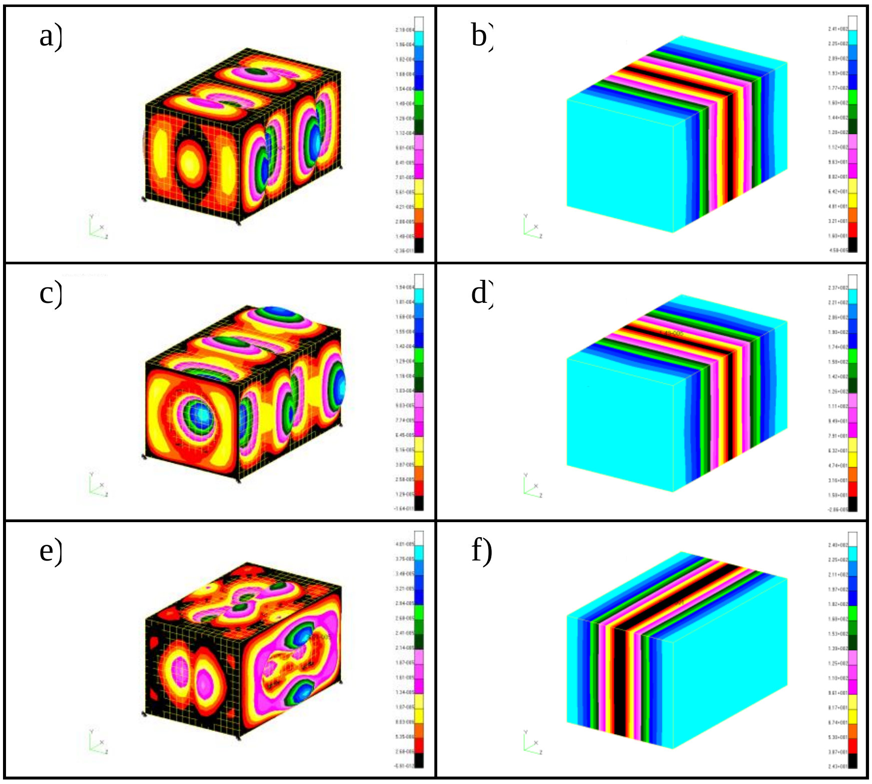

Fig. 9. Normal modes of a coupled box model:
a, b) at 68.352 Hz; c, d) at 71.848 Hz; and e, f) at 111.72 Hz.

In the second part of the graphs, above 69 Hz in Figs. 10 – 13, the FRF's demonstrate more complex behaviour. The maximum peaks in this part occur at different frequencies for each FRF. For example, in Fig. 12, the FRF at node 4826 has a maximum peak about 32 dB that occurs at 175 Hz, whereas for the FRF at node 6825, the maximum peak is about 27 dB at about 130 Hz. This means that one and the same excitation can affect in different way a potential receiver. In this case, the position of node 4826 is much more prone to higher interior noise than the position of node 6825. Of course, under different conditions the situation might be different.

As expected and as was mentioned above, the position of an external force can influence significantly the sound pressure response in a box structure. If a force acts close to a nodal line of a structural model, then the force can not excite many of the structural normal modes and the pressure response inside the model will be much lower in a certain frequency range. In practice, a complex geometry of the structure and a high dencity of the normal modes make it quite difficult to find the appropriate nodal lines. However, in the case of success, a noticeable noise reduction can be achieved. In this regard, the comparison between FRF's for different positions of the disturbing force, see Figs. 10 and 11, shows significant differences between the overall pressure responses. This means that the disturbing force applied to these positions excites different normal modes. Thus, the center of the bottom plate is an anti-nodal position for some of the normal modes, and a force applied to this position can induce a significant sound pressure response inside the box model. In the same time, the position in the vicinity of a corner of the bottom plate can be hardly considered as anti-nodal for whichever normal mode, and generation of interior sound by a disturbing force applied there can be substantially reduced.

Another key feature that can influence the sound pressure response in an enclosed cavity is the position of a receiver (e.g., a microphone). As in the case of external force, the position of a microphone can increase or decrease noise level perceived by a receiver. Assuming that the location of a measurement device is close to a nodal line of a certain acoustic mode of the enclosure, the pressure response at that position will be insignificant compared to all other positions.

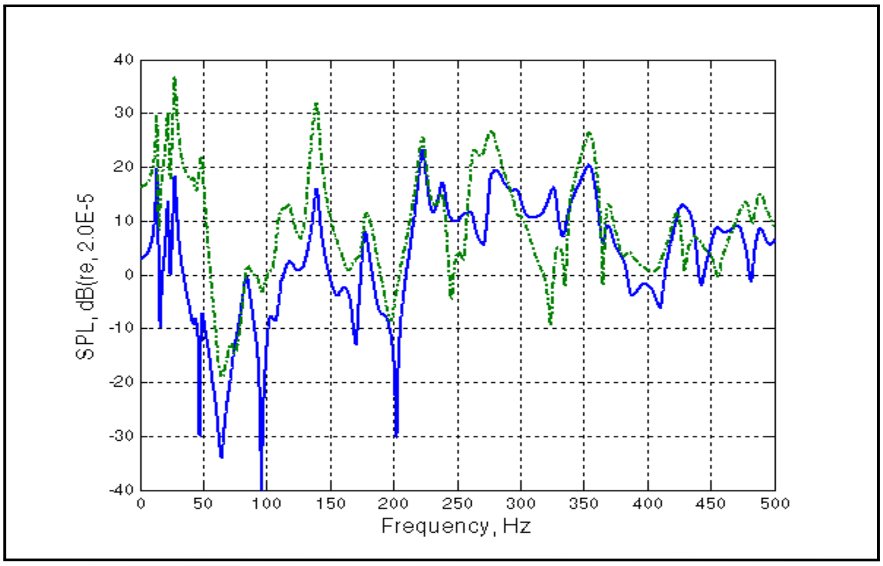

Fig. 10. Structural-acoustic FRF's calculated at node 4826 for a driving force applied close to a corner (solid curve) and in the centre of the bottom plate (dash-dotted curve)

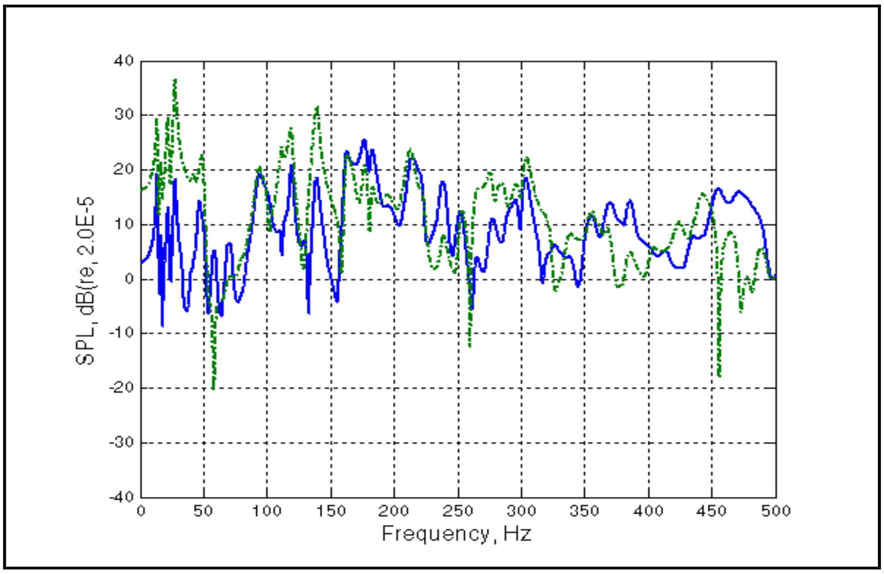

Fig. 11. Structural-acoustic FRF's calculated at node 6825 for a driving force applied close to a corner (solid curve) and in the centre of the bottom plate (dash-dotted curve)

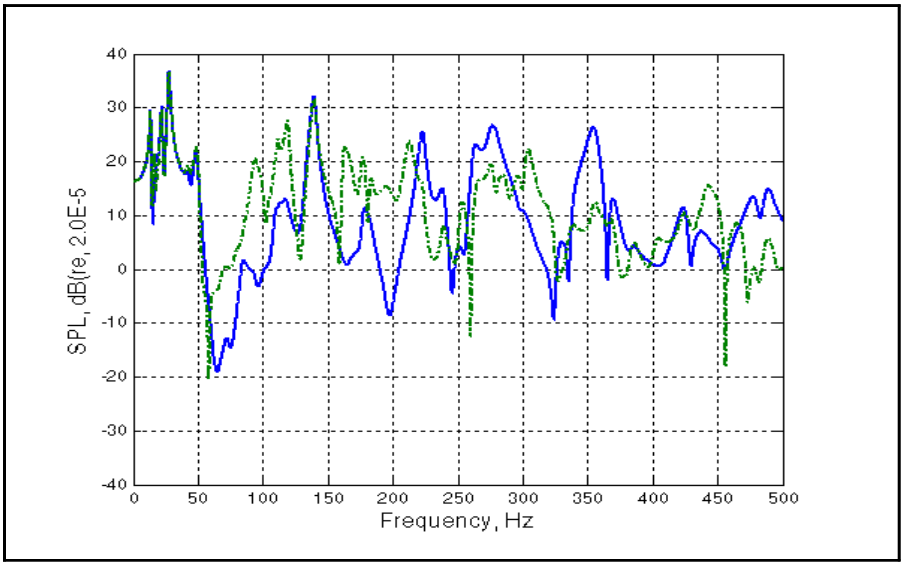

Fig. 12. Structural-acoustic FRF's calculated at node 4826 (solid curve) and at node 6825 (dash-dotted curve) for a driving force applied in the centre of the bottom plate

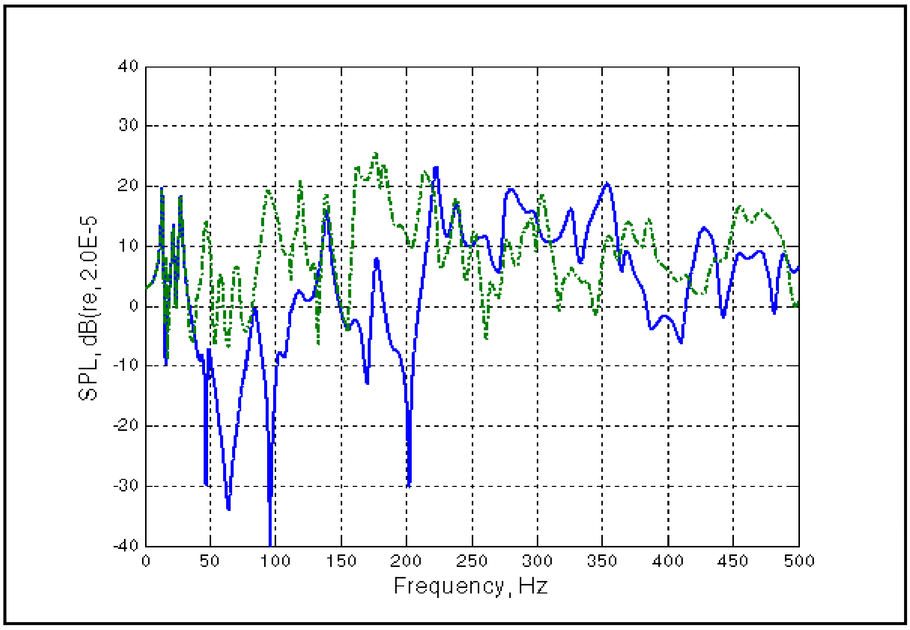

Fig. 13. Structural-acoustic FRF's calculated at node 4826 (solid curve) and at node 6825 (dash-dotted curve) for a driving force applied close to a corner of the bottom plate

In this regard, comparing the graphs shown in Figs. 12 and 13, one can notice that for a range between 50 and 200 Hz the FRF at node 4826 is significantly lower, in comparison with the pressure FRF at node 6825. Because node 4826 (the center of a rectangular enclosure) is nodal for the first four acoustic modes (see Fig. 6), the decrease in sound pressure level is well noticeable in the considered frequency range. Above 200 Hz, this location is not nodal any more, and the sound pressure level becomes nearly the same as that for node 6825.

4. Conclusions

In the present paper, a comprehensive finite element analysis of structural, acoustic, and coupled structural-acoustic properties of all-flexible rectangular boxes has been carried out. Although the all-flexible model is geometrically similar to the widely used simple model of a rigid rectangular box with only one flexible wall (that can be described analytically), it is much more complex from the point of view of its structural-acoustic behaviour. Therefore, its analysis represents an important step forward in understanding structural-acoustic properties of more complex and more realistic structural-acoustic models.

The initial attention in this study has been paid to the uncoupled structural behaviour of the model, where the results of the pioneering papers in this area have been revisited and their accuracy improved.

In the second part of the paper, a fully coupled structural-acoustic analysis has been undertaken, and a number of coupled structural-acoustic modes and a set of structural-acoustic frequency response functions have been calculated and analysed for different positions of a driving force and a receiver. The results obtained at different positions demonstrate that, depending on the position of a driving force and a receiver, the resulting frequency response functions can be significantly reduced in certain frequency intervals.

References

1. Dickinson, S.M., Warburton, G.B., "Vibration of box-type structures", *J. Mech. Eng. Sci*., vol. 9, pp. 325-334, 1967.

2. Hooker, R.J., O'Brien, D.J., "Natural frequencies of box-type structures by a finite element method", *Journal of Applied Mechanics,* vol. 41, pp. 363-365, 1974.

3. Grice, R.M., Pinnigton, R.J., "Vibration analysis of a thin-plate box using a finite element model which accommodates only in-plane motion", *Journal of Sound and Vibration*, vol. 232(2), pp. 449-471, 2000.

4. Grice, R.M., Pinnington, R.J., "Analysis of the flexural vibration of a thin-plate box using a combination of finite element analysis and analytical impedances", *Journal of Sound and Vibration*, vol. 249(3), pp. 499-527, 2002.

5. Fulford, R.A., Petersson B.A.T., "Estimation of vibrational power in built-up systems involving box-like structures, part 1: Uniform force distribution", *Journal of Sound and Vibration*, vol. 232(5), pp. 877-895, 2000.

6. Fulford, R.A., Petersson, B.A.T., "Estimation of vibrational power in built-up systems involving box-like structures, part 2: Infinite top-plate and circular geometry", *Journal of Sound and Vibration*, vol. 232(5), pp. 897-915, 2000.

7. Liang, J., Petersson, B.A.T., "Dominant dynamic characteristics of built-up structures", *Journal of Sound and Vibration*, vol. 247(4), pp. 703-718, 2001.

8. Petersson, B.A.T., "Vibro-acoustic features of box-like structures*", Proceedings of the International Conference "NOVEM 2005"*, Saint Raphael, France, April 2005, 12 pp. (on CD).

9. Marburg, S., Beer, H.J., Gier, J., Hardtke, H.J., Rennert, R., Perret, F., "Experimental verification of structural-acoustic modelling and design optimization", *Journal of Sound and Vibration*, vol. 252(4), pp. 591-615, 2002.

10. Hagiwara, I., Wang, D.W., Shi, Q.Z., Rao, R.S., "Reduction of noise inside a cavity by piezoelectric actuators", *Journal of Vibration and Acoustics*, vol. 125, pp. 12-17, 2003.

11. Luo, J., Gea, H.C., "Optimal stiffener design for interior sound reduction using a topology optimization based approach", *Journal of Vibration and Acoustics*, vol. 125, pp. 267-273, 2003.
12. Damaren, C.J., "Optimal location of collocated piezo-actuator/sensor combinations in spacecraft box structures", *Smart Materials and Structures*, vol. 12, pp. 494-499, 2003.
13. Georgiev, V.B., Krylov, V.V., "Numerical investigation of structure-borne interior noise in flexible rectangulaar boxes", *Proceedings of the Institute of Acoustics*, vol. 29(5), pp. 50-57, 2007.
14. Skudrzyk, E., *Simple and complex vibratory systems*, The Pennsylvania State University Press, 1968, pp 281-299.
15. Georgiev, V.B., Krylov, V.V., Winward, R.E.T.B., "Finite element analysis of structural-acoustic interaction in simplified models of road vehicles", *Proceedings of the Institute of Acoustics*, vol. 26(2), pp. 25-36, 2004.
16. Georgiev, V.B., Krylov, V.V., Winward, R.E.T.B., "Finite element calculations of structural-acoustic modes of vehicle interior for simplified models of motorcars", *Proceedings of the 33rd International Congress and Exposition on Noise Control Engineering, "Internoise 2004"*, Prague, Czech Republic, 22-25 August 2004, 8 pp. (on CD).
17. Krylov, V.V., Georgiev, V.B., "New approach to investigation of resonant vibrations of noncircular shells based on the theory of coupled waveguides", *Journal of Mechanics of Materials and Structures*, vol. 2(9), pp. 1761-1771, 2007.
18. Fahy, F.J. *Sound and structural vibration*. Academic Press, London, 1985.