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Torsional Wave Propagation in Carbon Nanotube Bundles

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Abstract

Torsional wave propagation in carbon nanotube bundle structures has been analyzed with Nonlocal Strain Gradient Theory. Governing equation of motion of carbon nanotube bundles have been derived. Phase and group velocity relations have been obtained. Elastic medium has been considered as a matrix material between the nanotubes. Effect of nonlocal stress and strain gradient parameters and stiffness of elastic medium to the torsional wave frequency and phase and group velocities have been investigated. Present results could be useful in designing of composite materials for vibration isolators.

Key words: carbon nanotubes, wave propagation, torsional waves, nonlocal strain gradient.

Распространение крутильных волн в пучках углеродных нанотрубок

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Аннотация

Распространение крутильных волн в структурах пучка углеродных нанотрубок было проанализировано с помощью Нелокальной теории градиента деформации. Получено управляющее уравнение движения пучков углеродных нанотрубок. Получены соотношения фазовых и групповых скоростей. Упругая среда рассматривалась как матричный материал между нанотрубками. Исследовано влияние параметров нелокального напряжения и градиента деформации, а также жесткости упругой среды на частоту крутильных волн, а также фазовые и групповые скорости. Представленные результаты могут быть полезны при проектировании композиционных материалов для виброизоляторов.

Ключевые слова: *углеродные нанотрубки, распространение волн, крутильные волны, градиент нелокальной деформации.*

Introduction

Carbon nanotubes (CNTs) have become a popular material for the last 20 years. Concept of designing a structure with superior properties have been getting attention of both industry and scientists. CNT bundles consist of N number of nanotubes which they can be wrapped to each other or can be embedded in a matrix material. CNT bundle structures could be used as nano-wires or nano-fibers in composites. Torsion in CNT bundles must be analyzed especially for the nano-wire applications of CNTs [1–3].

Nanoscale structures can be modeled with continuum theories. Differently from the macroscale mechanics, small scale effect can not be ignored in the nano-scale analysis. Strain [4, 5] and stress [6, 7] gradient nonlocal theories include the size effect and they have been used in most of the recent research about modeling of CNTs. Recently, Lim et al. [8, 9] proposed

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a nonlocal strain gradient model which considers both stress and strain gradient effects of an interval on a point in the continuum media.

Torsional characteristics of CNTs have been investigated by researchers over the years. Atomic simulation studies of ideal torsional properties of CNTs carried out in several studies [10–14]. Torsional vibration response of double-walled CNT structures [15], under the initial compression load [16] and buckyball attached to the free end [17] were studied. Theoretical modeling of torsional vibration of CNTs were obtained by using modified couple-stress theory [18], nonlocal elasticity theory [19] and strain gradient theory [20]. Torsional instability of CNTs has been also investigated in several papers [21–24]. Molecular dynamics modeling and analysis for torsional behavior of nanotubes [25–27] were studied comparatively with analytical results. Nonlocal torsional wave propagation in circular nanostructures [28] and multi-walled CNTs [29] were also investigated.

Li et al. [30, 31] pointed out that, both torsional enhancing and weakening effects in nonlocal theories are possible and correct. In a similar fashion, nonlocal strain gradient models were used in torsional wave propagation [32] and vibration [33, 34] of CNTs. Also newly developed nonlocal integral elasticity model has been used in analysis of torsional dynamics of CNTs [35, 36].

According to authors' best knowledge, torsional wave propagation in CNT bundle structures have not been investigated yet according to literature search. Therefore, torsional wave propagation in CNT bundle structures is studied using the nonlocal strain gradient theory. Elastic matrix material has been considered between the nanotubes in modeling. Wave propagation results are obtained for various parameters and velocities.

1. Analysis

1.1. Single CNT

A rod which has length (L) and diameter (d) is considered. The equation of motion in the angular direction can be written as [37]:

$$GI_P\frac{\partial^2\Theta}{\partial x^2} = \rho I_P\frac{\partial^2\Theta}{\partial t^2} + T \tag{1}$$

where G is the shear modulus, ρ is the density, I_P is the polar moment of inertia, Θ is the angular displacement of CNT and T is the distributed circumferential external torque. The I_P is defined as:

$$I_P = \pi\frac{(R_2^4 - R_1^4)}{2} \tag{2}$$

where R_1 and R_2 are the inner and outer radius of CNT respectively.

1.2. CNT Bundle

In CNT Bundle case, N number stacked nanotube which are embedded in an elastic matrix material assumed as shown in Fig. 1. The circumferential deformation of each nested tube is affected by elastic matrix material.

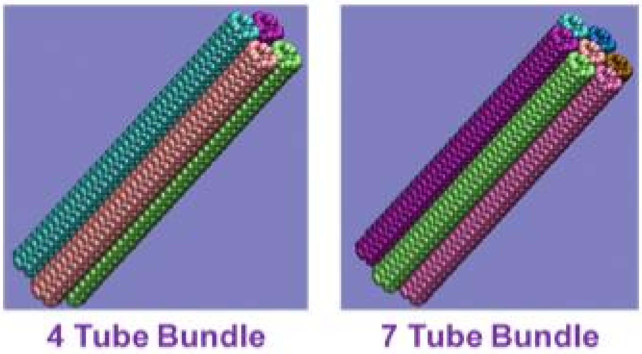

Fig. 1. CNT Bundle Structure [38]

The equations of motion of N number of tubes can be written by applying Eq. (1) for each tube:

$$T_1 = GI_P \frac{\partial^2 \Theta_1}{\partial x^2} - \rho I_P \frac{\partial^2 \Theta_1}{\partial t^2} \tag{3a}$$

$$T_i = GI_P \frac{\partial^2 \Theta_i}{\partial x^2} - \rho I_P \frac{\partial^2 \Theta_i}{\partial t^2} \tag{3b}$$

$$T_N = GI_P \frac{\partial^2 N}{\partial x^2} - \rho I_P \frac{\partial^2 \Theta_N}{\partial t^2} \tag{3c}$$

where Θ_i ($i = 1{,}2{,}...{,}N$) is the angular displacement of the i^{th} nanotube and the subscripts $1{,}2{,}...{,}N$ are used to denote the order of the nanotube. T_i is the total circumferential torque due to elastic matrix effect.

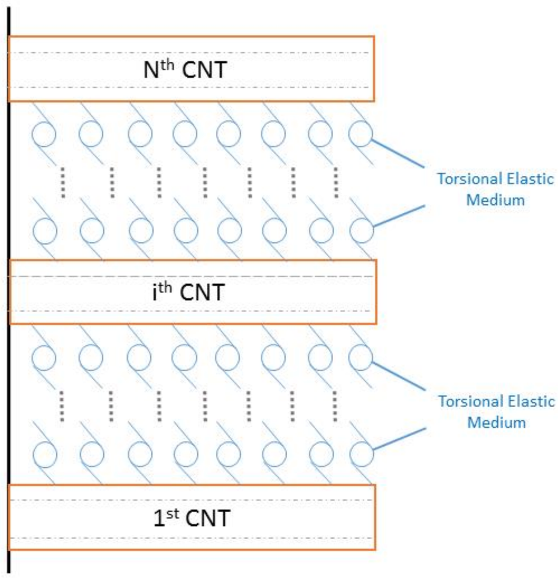

Fig. 2. Continuum Model of the One Dimensional CNT Bundle System

The torque relation due to elastic medium between nanotubes can be expressed as (Fig. 2):

$$T_1 = k(\Theta_1 - \Theta_2) \tag{4a}$$

$$T_i = k(2\Theta_i - \Theta_{i+1} - \Theta_{i-1}) \tag{4b}$$

$$T_N = k(\Theta_N - \Theta_{N-1}) \tag{4c}$$

where k is the stiffness of elastic matrix. CNT bundle system is consist of two carbon nanotubes with identical chirality's and they are covered with elastic medium (Fig. 1).

1.3. Nonlocal Strain Gradient Elasticity

The integral form of nonlocal stress relation can be written as [8, 39]:

$$\sigma_{ij} = C_{ijkl} \int_V \alpha_0(|x - x'|, e_0a)\varepsilon'_{kl}dV \tag{5a}$$

$$\sigma^{(1)}_{ijm} = (e_2a)^2 C_{ijkl} \int_V \alpha_1(|x - x'|, e_1a)\varepsilon'_{kl,m}dV \tag{5b}$$

where σ_{ij} is the nonlocal stress tensor, $\sigma^{(1)}_{ijm}$ is the high-order nonlocal stress tensor, e_0a and e_1a are nonlocal parameters which are related to nonlocal stress gradient field, e_2a is the material length scale parameter which is related to nonlocal strain gradient field. Material length scale parameters can be assumed as $e_0 = e_1$ for the rod type structures. $\alpha_0(|x - x'|, e_0a)$ and $\alpha_1(|x - x'|, e_1a)$ are the nonlocal kernel functions for the classical stress tensor and the higher order stress tensor, respectively. Nonlocal kernel functions satisfy the conditions in Eringen [39]. The nonlocal strain gradient theory states the total stress tensor accounts for both nonlocal and strain gradient tensors:

$$t_{ij} = \sigma_{ij} \pm \frac{\partial}{\partial x}\sigma^{(1)}_{ijm} \tag{6}$$

In. Eq. (6), sign of the higher order stress tensor can be assumed negative or positive. Generally, strain gradient models have stiffening effect on structure with negative higher order stress tensor. In the other hand, positive higher order stress tensor shows softening effect on strain gradient structure same as nonlocal stress gradient. Both parameters can affect the structure in stiffening or softening way depending on their negative or positive sign [40, 41]. Because of the lattice dynamics model of the elastic carbon nanotube structure predicts that travelling wave frequency in CNT structure decreases, softening strain gradient approach is used in the present study. Comparison of strain gradient rod models can be seen in Fig. 3.

1.4. Equation of Motion

The equation of motion and boundary conditions for torsional deformation of CNT are obtained using the Hamilton Principle and Nonlocal Strain Gradient Elasticity. The Hamilton Principle can be written as:

$$\int_{t_1}^{t_2} [\delta W + \delta E_K - \delta E_P]dt = 0 \tag{7}$$

where W denotes the work done by the elastic medium, E_K denotes the kinetic energy and E_P denotes the potential energy of the CNT. They are defined as [42, 43]:

$$W = \int_0^L T\Theta dx \tag{8a}$$

$$E_K = \int_0^L \rho I_P \left(\frac{\partial\Theta}{\partial t}\right)^2 dx \tag{8b}$$

$$E_P = \int_0^L GI_P \left(\frac{\partial\Theta}{\partial x}\right)^2 dx \tag{8c}$$

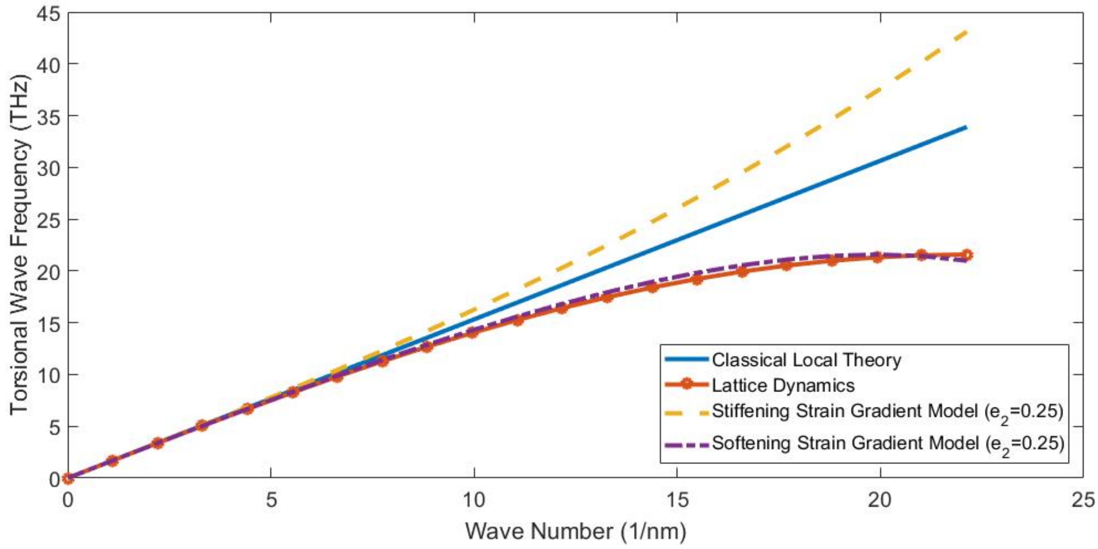

Fig. 3. Variation of Torsional Wave Frequency with Various Models

If W, E_K and E_P are defined according to the nonlocal strain gradient elasticity theory and variational principle, following equations are obtained:

$$\delta W = \int_{t_1}^{t_2}\int_0^L T\delta\Theta dxdt + \int_{t_1}^{t_2}\int_0^L \frac{\partial}{\partial x}(e_0 a)^2 \frac{\partial T}{\partial x}\delta\Theta dxdt \tag{9a}$$

$$\delta E_K = \int_0^L\int_{t_1}^{t_2} \frac{\partial}{\partial t}\left[\rho I_P\left(\frac{\partial\Theta}{\partial t}\right)\right]\delta\Theta dtdx + \int_{t_1}^{t_2}\int_0^L \frac{\partial}{\partial x}\left[(e_0 a)^2\rho I_P\left(\frac{\partial^3\Theta}{\partial x\partial t^2}\right)\right]\delta\Theta dxdt \tag{9b}$$

$$\delta E_P = \int_{t_1}^{t_2}\int_0^L \frac{\partial}{\partial x}\left[GI_P\left(\frac{\partial\Theta}{\partial x}\right)\right]\delta\Theta dxdt + \int_{t_1}^{t_2}\int_0^L \frac{\partial^2}{\partial x^2}\left[(e_2 a)^2 GI_P\left(\frac{\partial^2\Theta}{\partial x^2}\right)\right]\delta\Theta dxdt \tag{9c}$$

If Eq. (7) is rearranged according to Eqs. (9a)-(9c), Eq. (10) is obtained:

$$\left\{\int_{t_1}^{t_2}\int_0^L T\delta\Theta dxdt\right\} + \left\{-\int_{t_1}^{t_2}\int_0^L \frac{\partial}{\partial x}(e_0a)^2\frac{\partial T}{\partial x}\delta\Theta dxdt + \int_{t_1}^{t_2}\left[(e_0a)^2\frac{\partial T}{\partial x}\right][\delta\Theta(L)-$$

$$-\ \delta\Theta(0)]\,dt\} + \left\{-\int_0^L\int_{t_1}^{t_2}\frac{\partial}{\partial t}\left[\rho I_P\left(\frac{\partial\Theta}{\partial t}\right)\right]\delta\Theta dtdx + \int_0^L\left[\rho I_P\left(\frac{\partial\Theta}{\partial t}\right)\right][\delta\Theta(t_2)-\delta\Theta(t_1)]\,dx\right\}-$$

$$-\left\{\int_{t_1}^{t_2}\int_0^L\frac{\partial^2}{\partial x\partial t}\left[\mu\rho I_P\left(\frac{\partial^2\Theta}{\partial x\partial t}\right)\right]\delta\Theta dtdx - \int_{t_1}^{t_2}\left[\mu\rho I_p\left(\frac{\partial^3\Theta}{\partial x\partial t^2}\right)\right][\delta\Theta(L)-\delta\Theta(0)]\,dt\right\}-$$

$$-\left\{-\int_{t_1}^{t_2}\int_0^L\frac{\partial}{\partial x}\left[GI_P\left(\frac{\partial\Theta}{\partial x}\right)\right]\delta\Theta dtdx+\right.$$

$$+\int_{t_1}^{t_2}\left[GI_P\left(\frac{\partial\Theta}{\partial x}\right)\right][\delta\Theta(L)-\delta\Theta(0)]\,dt - \int_{t_1}^{t_2}\int_0^L\frac{\partial^2}{\partial x^2}\left[(e_2a)^2GI_P\left(\frac{\partial^2\Theta}{\partial x^2}\right)\right]\delta\Theta dtdx-$$

$$-\int_{t_1}^{t_2}\left[(e_2a)^2GI_P\left(\frac{\partial^2\Theta}{\partial x^2}\right)\right]\left[\frac{\partial\delta\Theta(L)}{\partial x}-\frac{\partial\delta\Theta(0)}{\partial x}\right]dt + \int_{t_1}^{t_2}\left[(e_2a)^2GI_P\left(\frac{\partial^3\Theta}{\partial x^3}\right)\right][\delta\Theta(L)-$$

$$-\ \delta\Theta(0)]\,dt\} = 0 \tag{10}$$

If Eq. (10) is reorganized, following equation is obtained:

$$\int_{t_1}^{t_2}\int_0^L\left\{T - \left[(e_0a)^2\left(\frac{\partial^2 T}{\partial x^2}\right)\right] - \left[\rho I_P\left(\frac{\partial^2\Theta}{\partial t^2}\right)\right] + \left[\mu\rho I_P\left(\frac{\partial^4\Theta}{\partial x^2\partial t^2}\right)\right] + \left[GI_P\left(\frac{\partial^2\Theta}{\partial x^2}\right)\right]+\right.$$

$$\left.+\ (e_2a)^2GI_P\left(\frac{\partial^4\Theta}{\partial x^4}\right)\right\}\delta\Theta dtdx + \int_{t_1}^{t_2}\left\{\left[(e_0a)^2\left(\frac{\partial T}{\partial x}\right)\right] - \left[\mu\rho I_P\left(\frac{\partial^3\Theta}{\partial x\partial t^2}\right)\right] - \left[GI_P\left(\frac{\partial\Theta}{\partial x}\right)\right]-\right.$$

$$\left.-\left[(e_2a)^2GI_P\left(\frac{\partial^3\Theta}{\partial x^3}\right)\right]\right\}[\delta\Theta(L)-\delta\Theta(0)]\,dt + \int_{t_1}^{t_2}\left[(e_2a)^2GI_P\left(\frac{\partial^2\Theta}{\partial x^2}\right)\right]\left[\frac{\partial\delta\Theta(L)}{\partial x}-\right.$$

$$\left.-\ \frac{\partial\delta\Theta(0)}{\partial x}\right]dt = 0 \tag{11}$$

According to Eq. (11), the governing equation of motion of a CNT can be written as:

$$GI_P\left(\frac{\partial^2\Theta}{\partial x^2}\right) + (e_2a)^2GI_P\left(\frac{\partial^4\Theta}{\partial x^4}\right) = \rho I_P\left(\frac{\partial^2\Theta}{\partial t^2}\right) - (e_0a)^2\rho I_P\left(\frac{\partial^4\Theta}{\partial x^2\partial t^2}\right) + T- -(e_0a)^2\left(\frac{\partial^2 T}{\partial x^2}\right) \tag{12}$$

and the boundary conditions are obtained as:

$$\left[(e_0a)^2\left(\frac{\partial T}{\partial x}\right) - \mu\rho I_P\left(\frac{\partial^3\Theta}{\partial x\partial t^2}\right) - GI_P\left(\frac{\partial\Theta}{\partial x}\right) - (e_2a)^2GI_P\left(\frac{\partial^3\Theta}{\partial x^3}\right)\right][\delta\Theta] = 0 \tag{13a}$$

$$\left[-(e_2a)^2GI_P\left(\frac{\partial^2\Theta}{\partial x^2}\right)\right]\left[\frac{\partial\delta\Theta}{\partial x}\right]=0 \tag{13b}$$

Eq. (12) is the governing equation of the nonlocal strain gradient CNT for the torsional deformation. If the nonlocal parameter is assumed as zero ($e_0 = 0$), the strain gradient rod model is obtained. If the strain gradient parameter is assumed as zero ($e_2 = 0$), the nonlocal rod model is obtained. If the nonlocal and strain gradient parameters are assumed as zero ($e_0 = e_2 = 0$), classical rod model is obtained. If Eqs. (3) and (4) are inserted into Eq. (12), the equations of motion for CNT bundle is obtained as:

$$\begin{gathered}GI_P\frac{\partial^2\Theta_1}{\partial x^2}+(e_2a)^2GI_P\frac{\partial^4\Theta_1}{\partial x^4}=\rho I_P\frac{\partial^2\Theta_1}{\partial t^2}-(e_0a)^2\rho I_P\frac{\partial^4\Theta_1}{\partial x^2\partial t^2}+k(\Theta_1-\Theta_2)-\\-(e_0a)^2k\left(\frac{\partial^2\Theta_2}{\partial x^2}-\frac{\partial^2\Theta_1}{\partial x^2}\right)\end{gathered} \tag{14a}$$

$$\begin{gathered}GI_P\frac{\partial^2\Theta_i}{\partial x^2}+(e_2a)^2GI_P\frac{\partial^4\Theta_i}{\partial x^4}=\rho I_P\frac{\partial^2\Theta_i}{\partial t^2}-(e_0a)^2\rho I_P\frac{\partial^4\Theta_i}{\partial x^2\partial t^2}+k(2\Theta_{(i)}-\Theta_{(i+1)}-\\-\Theta_{(i-1)})-(e_0a)^2k\left(2\frac{\partial^2\Theta_{(i)}}{\partial x^2}-\frac{\partial^2\Theta_{(i+1)}}{\partial x^2}-\frac{\partial^2\Theta_{(i-1)}}{\partial x^2}\right)\end{gathered} \tag{14b}$$

$$\begin{gathered}GI_P\frac{\partial^2\Theta_N}{\partial x^2}+(e_0a)^2GI_P\frac{\partial^4\Theta_N}{\partial x^4}=\rho I_P\frac{\partial^2\Theta_N}{\partial t^2}-(e_2a)^2\rho I_P\frac{\partial^4\Theta_N}{\partial x^2\partial t^2}+k(\Theta_{(N)}-\Theta_{(N-1)})-\\-(e_0a)^2k\left(\frac{\partial^2\Theta_{(N)}}{\partial x^2}-\frac{\partial^2\Theta_{(N-1)}}{\partial x^2}\right)\end{gathered} \tag{14c}$$

For the harmonic torsional wave propagation, displacement of each tube can be written as:

$$\Theta_i(x,t)=A_ie^{j(\omega t-mx)} \tag{15}$$

where ω is the torsional wave frequency, m is circumferential wave number and $j^2=-1$. Inserting Eq. (15) into Eqs. (14a)-(14c) leads to:

$$\begin{bmatrix}P_{11} & P_{12} & P_{13} & \dots & \dots & \dots & P_{1i}\\ P_{21} & P_{22} & P_{23} & \dots & \dots & \dots & P_{2i}\\ \cdot & \cdot & \cdot & & \cdot & & \cdot\\ \cdot & \cdot & \cdot & & \cdot & & \cdot\\ \cdot & \cdot & \cdot & & \cdot & & \cdot\\ \cdot & \cdot & \cdot & & \cdot & & \cdot\\ P_{N1} & P_{N2} & P_{N3} & \dots & \dots & \dots & P_{NN}\end{bmatrix}\begin{bmatrix}A_1\\ A_2\\ \cdot\\ \cdot\\ \cdot\\ A_{N-1}\\ A_N\end{bmatrix}=0 \tag{16}$$

where ψ_i is the amplitude of the i^{th} tube and related terms are defined in Eqs. (17a)-(17c):

$$\begin{gathered}P_{11}=m^4(e_2a)^2GI_P-m^2(GI_P-(e_0a)^2\rho I_P\omega^2+(e_0a)^2k)+(\rho I_P\omega^2-k)\\ P_{12}=m^2(e_0a)^2k+k\\ P_{13}=P_{14}=\dots=P_{1i}=0\end{gathered} \tag{17a}$$

$$P_{i(i-1)} = P_{i(i+1)} = m^2(e_0a)^2k + k$$

$$P_{ii} = m^4(e_2a)^2GI_P - m^2(GI_P - (e_0a)^2\rho I_P\omega^2 + 2(e_0a)^2k) + (\rho I_P\omega^2 - 2k) \tag{17b}$$

$$P_{i1} = ... = P_{i(i-3)} = P_{i(i-2)} = P_{i(i+2)} = P_{i(i+3)} = ... = P_{iN} = 0$$

$$P_{NN} = m^4(e_2a)^2GI_P - m^2(GI_P - (e_0a)^2\rho I_P\omega^2 + (e_0a)^2k) + (\rho I_P\omega^2 - k)$$

$$P_{(N-1)N} = m^2(e_0a)^2k + k \tag{17c}$$

$$P_{N1} = P_{N2} = P_{N(N-2)}... = 0$$

The determinant of the coefficient matrix in Eq. (16) must be equal to zero. If the determinant equation is solved for ω, the torsional wave frequencies for N-nanotube bundle system are obtained.

Phase velocity (ν_P) is the velocity of an individual particle which propagates in the structure and it is related only with the wavenumber, not any physical quantity (Eq. (18a)). Group velocity (ν_G) defines overall shape of the propagation of a group of waves at similar frequency and can be obtained using Eq. (18b).

$$\nu_P = \frac{\omega}{k} \tag{18a}$$

$$\nu_G = \frac{d\omega}{dk} \tag{18b}$$

2. Numerical Results and Discussion

In this section, validation of present model has been achieved, firstly. After that, variation of torsional wave frequency and phase and group velocities with various parameters have been investigated for the N=10 number of CNTs.

Numerical results for the torsional wave frequency analysis are obtained by assuming material constants: $G = 0{,}46TPa$, $\rho = 4962kg/m^3$. Various studies can be found about the determination of elastic properties and effective wall thickness of nanotubes. Inner radius of CNTs is chosen as 0,68 nm and thickness of CNT is accepted as 0,132 nm, respectively [44, 45].

An atomic lattice model for torsional wave propagation in SWCNT was proposed in previous study [37]. Frequency equations for Lattice Dynamic and Nonlocal Strain Gradient theories can be obtained as below:

$$\omega_{LD} = 2\frac{c}{a}\sqrt{\sin^2\left(\frac{ma}{2}\right)} \tag{19a}$$

$$\omega_{NLSG} = cm\sqrt{\frac{1-(e_2a)^2m^2}{1+(e_0a)^2m^2}} \tag{19b}$$

where c is the shear speed of sound ($c = \sqrt{G/\rho}$).

Variation of torsional wave frequency with wave number is seen in Fig. 4. The local frequency increases linearly with wave number. According to Lattice Dynamics, a travelling wave has a limit propagation velocity. The strain gradient ($e_2 = 0{,}25$) and nonlocal ($e_0 = 0{,}39$)

models show good agreement with the lattice dynamics results for the end of first Brillouin-Zone. Nonlocal strain gradient model gives almost identical results with lattice dynamics model for the selected parameters ($e_0 = 0{,}20$, $e_2 = 0{,}21$).

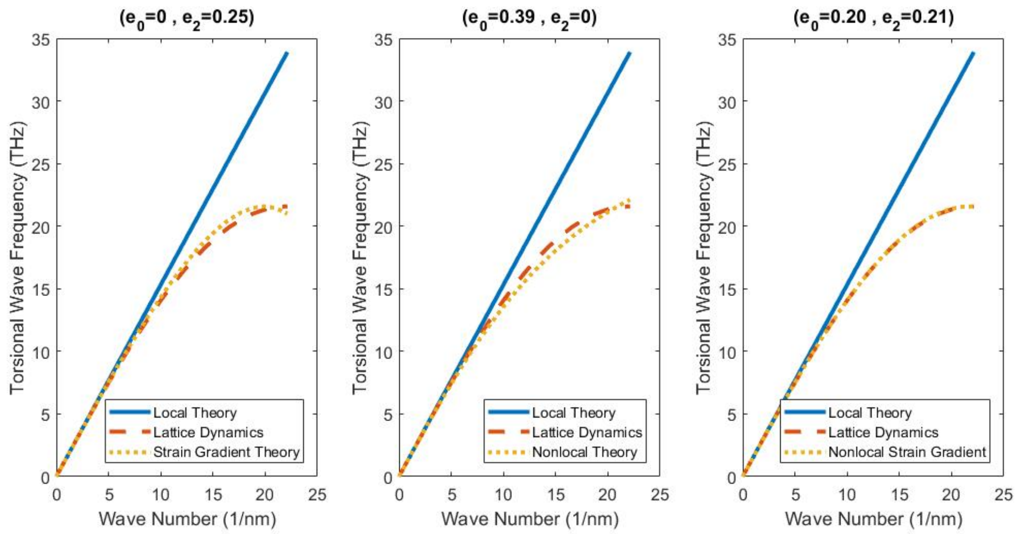

Fig. 4. Variation of Torsional Wave Frequency with Wave Number for Various Models

Effect of the elastic medium in torsional wave frequency is seen in Figs. 5 and 6. For the 1st CNT, elastic medium has no effect on torsional wave frequency. With the increasing number of CNTs, elastic medium becomes more effective and torsional wave frequency raises. Effect of the number of carbon nanotubes and elastic medium are more pronounced in small wave numbers (long wavelengths).

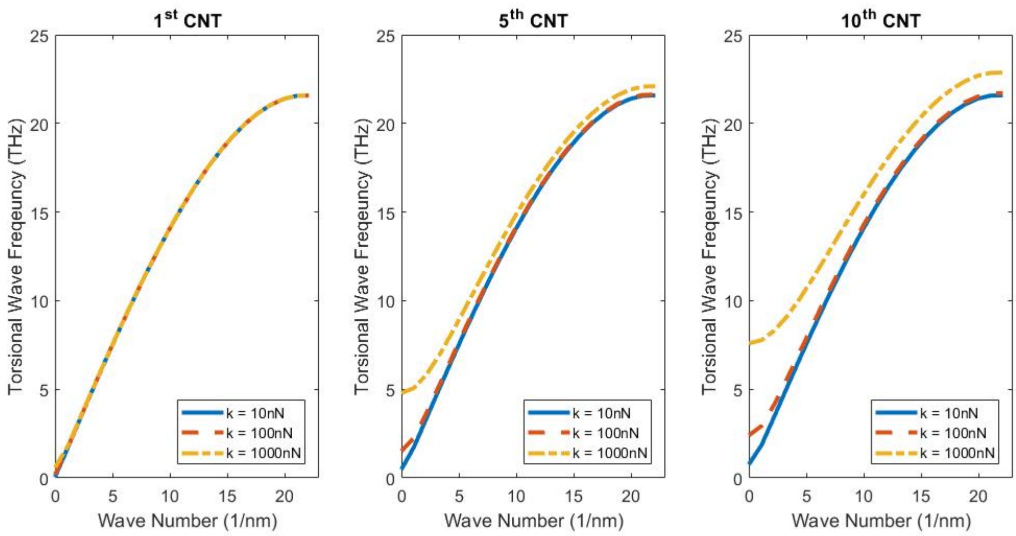

Fig. 5. Effect of Wave Number on Torsional Wave Frequency for Various Elastic Medium Stiffness's

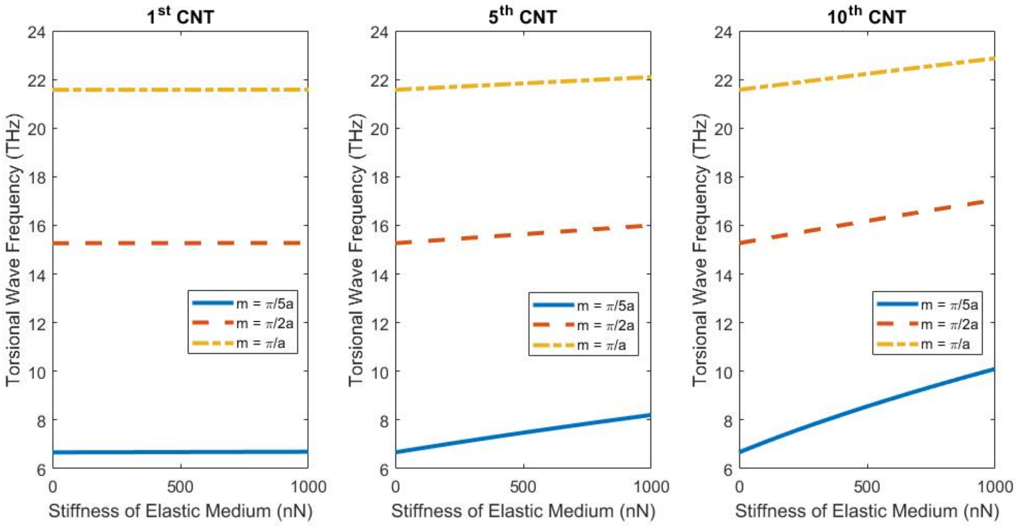

Fig. 6. Effect of Elastic Medium on Torsional Wave Frequency for Various Wave Numbers

In Figs. 7 and 8, variation of phase and group velocities for the 1st and 10th CNT can be seen. Number of CNT increases both phase and group velocity. Elastic medium is effective only in small interval at low frequencies (long wavelengths) for the 1st CNT. With the increasing number of CNT, effective frequency interval of group velocity is expanded. Elastic medium effect vanishes at high wave numbers (short wavelengths), because of the nonlocal strain gradient model. Phase and group velocities has not been affected by elastic medium at the end of first Brillouin Zone and show identically same characteristics.

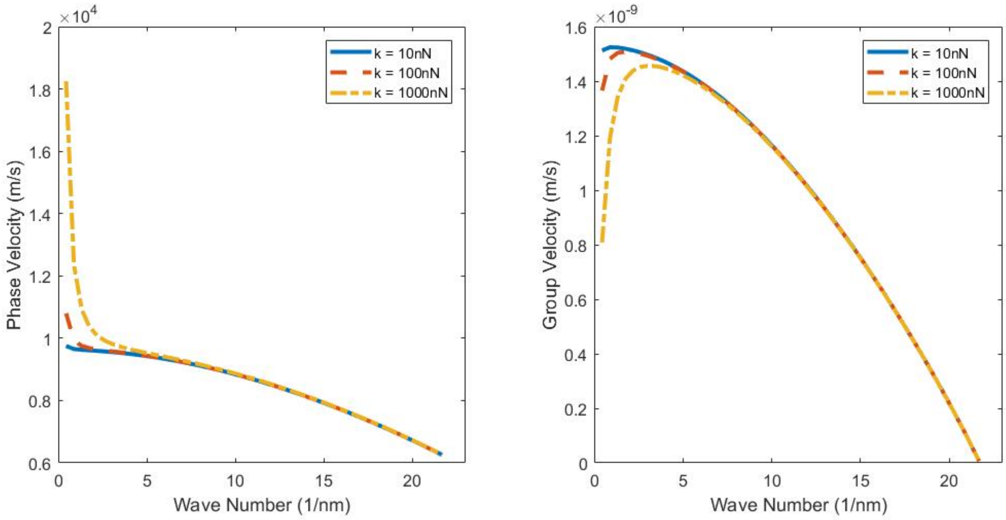

Fig. 7. Variation of Phase and Group Velocity of 1st CNT

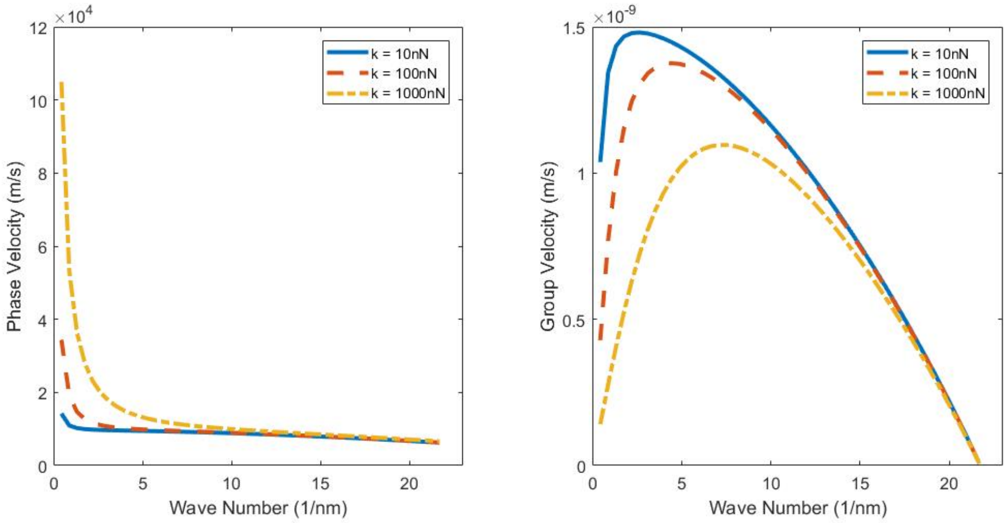

Fig. 8. Variation of Phase and Group Velocity of 10^{st} CNT

Conclusion

Torsional wave propagation in multiple CNTs stacked in an elastic matrix which is called CNT bundle has been investigated in the present study. Governing equation of motion has been obtained using Nonlocal Strain Gradient Elasticity Theory Torsional wave propagation frequencies, phase and group velocities for first and last CNTs have been determined. Effects of gradient parameters and stiffness of elastic medium have been investigated comparatively.

The nonlocal strain gradient elasticity model is more acceptable for CNTs rather than the only stress or strain gradient and classical theories. Elastic medium has more pronounce effect on wave frequency especially for increasing number of nanotubes. Group velocity effectiveness expands with increasing elastic medium stiffness and number of nanotubes.

Present results may be useful for modeling of composite materials for vibration isolators.

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