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Torsional vibration of double CNT system embedded in an elastic medium

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Abstract

The torsional vibration analysis of double carbon nanotube system (CNT system) is carried out in the present work. Carbon nanotubes are connected to each other with elastic matrix material. Eringen's Nonlocal Elasticity Theory is used in modeling of the system. The effects of nonlocal parameter and stiffness of elastic medium to the non-dimensional frequencies of the system are investigated in detail. Two frequency set are obtained for double carbon nanotube system for a given half wave number. It is also shown that some mode shapes are anti-phase and some of them are in-phase. The present results can be useful in design of nano-electromechanical systems like rotary servomotors.

Key words: torsional vibration, double carbon nanotube, nonlocal elasticity, elastic medium.

Торсионная вибрация двойной системы УНТ, встроенной в упругую среду

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Аннотация

В настоящей работе проведен анализ крутильных колебаний двойной углеродной нанотрубки (УНТ). Углеродные нанотрубки соединены друг с другом упругим материалом. При моделировании системы используется теория нелокальной упругости Эрингена. Подробно исследуются эффекты нелокальности и жесткости упругой среды на безразмерные частоты системы. Получены два набора частот для двойной углеродной нанотрубки для заданного полуволнового числа. Также показано, что некоторые моды являются антифазными, а некоторые из них являются синфазными. Настоящие результаты могут быть использованы при проектировании наноэлектромеханических систем, таких как вращающиеся сервомоторы.

Ключевые слова: *крутильная вибрация, двойная углеродная нанотрубка, нелокальная упругость, упругая среда.*

Introduction

Carbon nanotubes (CNTs), which was discovered by Iijima [1], have been a very popular material for scientists and industry. Superior physical properties of carbon nanotubes have paved the way of applications that seems impossible before [2]. Nowadays, engineers have been designing probable applications for CNTs in sensor technologies, nano-mechanical components, electromechanical systems, etc.

Two main approaches have been used in the CNT modeling: discrete and continuum models. Discrete models are based on interactions in atomic lattice structure. Molecular Dynamics Simulation and Lattice Dynamics are discrete models. Continuum models can also use in modeling of CNTs. But, the classical continuum mechanics approach is not suitable at the nano length scale due to its intrinsic length free formulation. Unlike the macroscale

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mechanics, small scale effect and long distance interaction can not be ignored in nano-dimensional mechanics. Nonlocal Elasticity Theory, which includes size effect, was proposed by Eringen [3–5]. With this assumption, he combined discrete and continuum models into one model.

Peddision et al. [6] employed the nonlocal elasticity theory and obtained the nonlocal Euler-Bernoulli beam model, firstly. Wang and Varadan [7] studied the wave propagation characteristics of CNTs. Duan et al. [8] calibrated the small scale parameter of the nonlocal Timoshenko beam theory according to MD simulation results. Wang and Wang [9] presented the constitutive relations of nonlocal elasticity theory for Euler–Bernoulli, Timoshenko and cylindrical shells theories. Reddy [10] reformulated various beam theories, including the Euler–Bernoulli, Timoshenko, Reddy and Levinson beam theories, using the nonlocal differential constitutive relations. Aydogdu [11] proposed a generalized nonlocal beam theory for bending, buckling and free vibration of nanobeams. Gupta et al. investigated the vibration [12] and wall thickness and elastic moduli [13] of single-walled carbon nanotubes (SWCNTs).

Torsional behaviour of CNTs has taken interests of the researchers in recent years. Possible application areas of CNTs have been reported by scientist as torsional oscillator [14, 15], nano-electromechanical devices like biological rotary nano-servomotors [16–19] and torsion sensor in nano-composites [20]. Wang et al. [21] modeled torsional deformation of carbon nanotubes with using atomistic simulation. Ertekin and Chrzan [22] investigated the ideal torsional strength and stiffness of carbon nanotubes. Hall et al. [23] made an experimental measurements of SWCNT's torsional properties. Liang and Upmanyu [24] showed the relation between torsion and axial deformation in CNT. Zhang and Wang [25] investigated the torsional buckling response of double-walled carbon nanotubes (DWCNTs) with using MD simulation results. Gheshlaghi et al. [26] used the modified couple stress theory for the torsional vibration analysis of CNTs. Vercosa et al. [27] studied the torsional instability of carbon nanotubes. Murmu et al. [28] modeled a mass sensor system which consists of CNT and fullerene. Li et al. [29–31] proposed a semi-continuum model which considers the both nonlocal softening and enhancing effects. Strain gradient [32], nonlocal stress gradient [33] and moleculer dynamics simulation [34] of torsional vibration of CNTs studied by researchers. Demir ve Civalek [35] investigated the size effects in the torsional and axial response of microtubules. Kiani [36] studied the longitudinal, transverse, and torsional vibrations and stabilities of axially moving SWCNTs. Molecular dynamics study of boron-nitride nanotubes was carried out by Ansari and Ajori [37]. Torsional vibration of CNTs embedded in an elastic medium [38] and viscoelastic medium [39], torsional vibration of DWCNTs [40], torsional wave propagation in MWCNTs [41] and nonlocal strain gradient analysis of torsional vibration and wave propagation of CNTs [42, 43] were carried out by Arda and Aydogdu. Torsional vibration of CNTs with axial velocity gradient effect studied by Guo et al. [44]. Fatahi-Vajari and Imam [45] used the doublet mechanics theory in torsional vibration analysis of CNTs. Zhu and Li [46] used nonlocal integral elasticity approach in longitudinal and torsional vibrations of size-dependent rods. An enhanced form of nonlocal elasticity was used in torsional vibration of nanobeams by Apuzzo et al. [47] Torsional vibration of bi-directional functionally graded nanotubes studied by Li and Hu [48].

Murmu et al. has published some papers about longitudinal [49, 50] and flexural [51, 52] vibration of double CNT systems. According to author's knowledge, torsional vibration of double carbon nanotube (DCNT) system embedded in an elastic medium has not been considered yet. The aim of this study is to investigate the torsional dynamics of the DCNT system considering nonlocality and stiffness of elastic medium. Effect of the parameters to the DCNT system's mode shapes are depicted.

1. Analysis

Let's assume a carbon nanotube with length L and diameter d. The stress resultant for the nanotube due to the shear stress is expressed as:

$$S = \int_A \tau \, dA \tag{1}$$

where A is the cross-section area of the CNT, and the torque relation is given as:

$$T = \int_A \tau z \, dA \tag{2}$$

where z is a distance from center of the circular section. The equation of motion for torsional deformation is expressed as [53]:

$$GI_P \frac{\partial^2 \theta}{\partial x^2} = \rho I_P \frac{\partial^2 \theta}{\partial t^2} + T \tag{3}$$

where ρ is the density, I_P is the polar moment of inertia, R_1 and R_2 is the inner and outer radius, θ is the angular displacement of CNT and T is the elastic medium torque effect. The I_p is defined as:

$$I_P = \pi \frac{({R_2}^4 - {R_1}^4)}{2} \tag{4}$$

2. Double CNT System

Double carbon nanotube system is consist of two carbon nanotubes with identical chiralites and they are covered with elastic medium (Fig. 1).

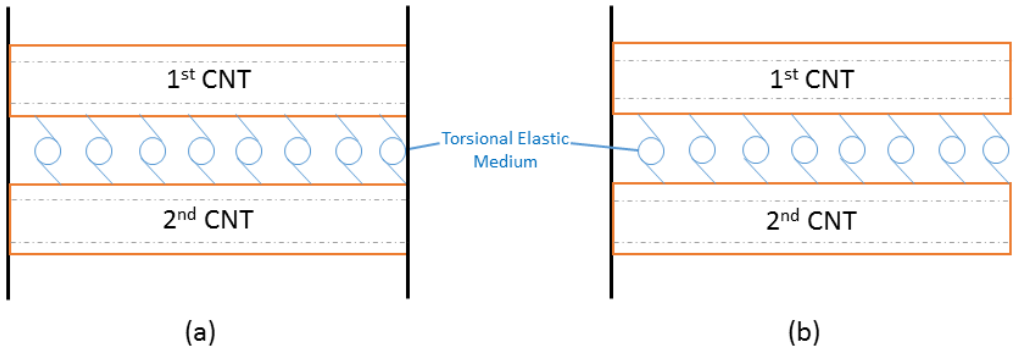

Fig. 1. Double CNT System with Elastic Medium:
(a) C-C and (b) C-F Boundary Conditions

Considering the elastic medium effect between the two tubes, the equations of motion of each tubes can be written as:

$$G_i {I_P}_i \frac{\partial^2 \theta_i}{\partial x^2} = \rho_i {I_P}_i \frac{\partial^2 \theta_i}{\partial t^2} + T_i \tag{5}$$

where subscripts i is used to define the quantities belongs to tube. θ_i is the angular displacement, Ip_i is the polar moment of inertia and G_i is the shear modulus of the corresponding tube. T_i is the torque that occurred by interaction due to elastic medium. Elastic medium effect on first and second CNTs are defined as below:

$$T_1 = k(\theta_1 - \theta_2) \tag{6}$$

$$T_2 = k(\theta_2 - \theta_1) \tag{7}$$

where k is the stiffness of the elastic medium which covers the CNTs.

3. Nonlocal Elasticity Theory

The nonlocal constitute relation can be given as [3, 4, 11];

$$(1 - \mu \nabla^2)\tau_{kl} = \lambda \varepsilon_{rr} \delta_{kl} + 2G\varepsilon_{kl} \tag{8}$$

where τ_{kl} is the nonlocal stress tensor, ε_{kl} is the strain tensor, λ and G are the Lame constants, $\mu=(e_0a)^2$ is called the nonlocal parameter, a is an internal characteristic length and e_0 is a constant. Eringen [3] determined this parameter with matching the dispersion curves based on the atomic models. Wang et al [54] made estimation for the SWCNT as $e_0a \leq 2nm$. Aydogdu [55] has obtained that e_0 is material and length dependent for axial wave propagation.

For the torsional deformation of uniform CNT, Eq. (8) can be written in the one dimensional form:

$$\left(1-\mu\frac{\partial^2}{\partial x^2}\right)\tau = G\gamma \tag{9}$$

where γ is the shear strain and τ is the shear stress of CNT. By using the Eq. (1), (2) and Eq. (9) we get the constitute relation as:

$$S-(e_0a)^2\frac{\partial^2 S}{\partial x^2} = GA\gamma \tag{10}$$

$$T-(e_0a)^2\frac{\partial^2 T}{\partial x^2} = GI_P\frac{\partial\theta}{\partial x} \tag{11}$$

If Eq. (11) is inserted into Eq. (3) one obtains:

$$GI_P\frac{\partial^2\theta}{\partial x^2} = \left(1-\mu\frac{\partial^2}{\partial x^2}\right)\rho I_P\frac{\partial^2\theta}{\partial t^2}+\left(1-\mu\frac{\partial^2}{\partial x^2}\right)T \tag{12}$$

Eq. (12) is the governing equation of the CNT for the torsional deformation. If we choose μ=0 we get the classical elasticity equation of torsional deformation. If Eq. (6) and Eq. (7) are inserted into Eq. (12), the equations of motion for DWCNT are obtained as:

$$GI_P\frac{\partial^2\theta_1}{\partial x^2} = \rho I_P\frac{\partial^2\theta_1}{\partial t^2}-\mu\rho I_P\frac{\partial^4\theta_1}{\partial x^2\partial t^2}+k(\theta_1-\theta_2)-\mu k\left(\frac{\partial^2\theta_1}{\partial x^2}-\frac{\partial^2\theta_2}{\partial x^2}\right) \tag{13}$$

$$GI_P\frac{\partial^2\theta_2}{\partial x^2} = \rho I_P\frac{\partial^2\theta_2}{\partial t^2}-\mu\rho I_P\frac{\partial^4\theta_2}{\partial x^2\partial t^2}+k(\theta_2-\theta_1)-\mu k\left(\frac{\partial^2\theta_2}{\partial x^2}-\frac{\partial^2\theta_1}{\partial x^2}\right) \tag{14}$$

For harmonic vibration, the angular displacement θ_i can be expressed as:

$$\theta_i(x,t) = \psi_i(x)e^{i\omega t} \tag{15}$$

where ω is the angular velocity. To find simple analytical solutions for the Clamped-Clamped (C-C) and Clamped-Free (C-F) boundary conditions, $\psi(x)$ can be assumed as:

$$\psi_i(x) = A_i\sin(\beta x) \tag{16}$$

where A_i is the amplitude of the i^{th} tube. β is the characteristic parameter and can be defined as $\beta = m\pi$ for (C-C) boundary condition and $\beta = \frac{2m-1}{2}\pi$ for (C-F) boundary condition where m is the half wave number. If we insert Eq. (20) into Eq. (18) and Eq. (19) with dimensionless parameter ($\bar{x} = \frac{x}{L}$) we get the following dimensionless equations of motion:

$$\frac{\partial^2\theta_1}{\partial\bar{x}^2}\left(1-\frac{\mu}{L^2}\Omega^2+\frac{\mu}{L^2}K\right)+\theta_1(\Omega^2-K)+\frac{\partial^2\theta_2}{\partial\bar{x}^2}\left(-\frac{\mu}{L^2}K\right)+\theta_2(K) = 0 \tag{17}$$

$$\frac{\partial^2\theta_1}{\partial\bar{x}^2}\left(-\frac{\mu}{L^2}K\right)+\theta_1(K)+\frac{\partial^2\theta_2}{\partial\bar{x}^2}\left(1-\frac{\mu}{L^2}\Omega^2+\frac{\mu}{L^2}K\right)+\theta_2(\Omega^2-K) = 0 \tag{18}$$

where related terms are defined

$$\Omega^2 = \frac{\rho\omega^2L^2}{G}\quad,\quad K = \frac{kL^2}{GI_P} \tag{19}$$

where Ω is the non-dimensional frequency parameter (NDFP) and K is the non-dimensional stiffness of elastic medium. Introducing Eq. (16) into Eq. (17) and Eq. (18) gives following eigen-value equation:

$$\begin{bmatrix}-\beta^2\left(1-\frac{\mu}{L^2}\Omega^2+\frac{\mu}{L^2}K\right)+(\Omega^2-K) & \beta^2\frac{\mu}{L^2}K+K\\ \beta^2\frac{\mu}{L^2}K+K & -\beta^2\left(1-\frac{\mu}{L^2}\Omega^2+\frac{\mu}{L^2}K\right)+(\Omega^2-K)\end{bmatrix}\begin{bmatrix}A_1\\A_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} \tag{20}$$

Non-dimensional frequencies for the double CNT system can be obtained from the determinant of the coefficient matrix in Eq. (20). It should be noted that for a given half wave number m, two frequencies are obtained: Ω_L is the lower order resonant frequency and Ω_H is higher order resonant frequency. Inserting the NDFP into Eq. (20) gives the amplitude ratio of the two tubes in the following form:

$$\frac{A_2}{A_1} = \frac{-\beta^2\left(1-\frac{\mu}{L^2}\Omega^2+\frac{\mu}{L^2}K\right)+\left(\Omega^2-K\right)}{\beta^2\frac{\mu}{L^2}K+K} \quad (21)$$

From the Eq. (21) it can be seen that, amplitude ratio may be positive or negative. The positive ratio means an in-phase motion in which nanotubes rotate in same direction and negative ratio means anti-phase motion in which nanotubes rotate in opposite directions. In the next section, mode shapes are shown in detail.

4. Numerical Results and Discussion

In this section, the NDFPs of torsional vibration of DCNT system are investigated for various nonlocal parameter and elastic medium stiffness parameter.

There have been many researches about physical properties of CNTs. Nanotube radius has essential role on the shear modulus (*G*). In the present study, it is selected from the [56]. Density (ρ) of CNTs is determined using the calculation method given in [57]. There have been different assumptions in literature about thickness of CNT. In this work, CNT thickness is accepted as 0.132 nm according to Ref. [58]. Material properties of CNTs are given in Table 1.

Table 1
Material properties for CNT

CNT	Inner Radius (R_i) (nm)	Density (ρ) (kg/m^3)	Shear Modulus (G) (TPa)
Armchair (6,6)	0.409	4961	0.425

Validation of the present nonlocal nanotube model has been carried out in previous studies [38, 40, 41]. Two different discrete model (Lattice and Molecular Dynamics) torsional frequency results have been used in order to compare the stress gradient nonlocal model. Nonlocal theory results are in good agreement with discrete model results.

5. Results

Nonlocal effect on DCNT system’s non-dimensional frequencies (NDF) can be seen in Fig. (2). Nonlocality decreases the both higher and lower order frequencies with softening effect in lattice structure. Nonlocal effect is less effective in (C-F) case because of the geometric condition at the free end. Stiffness of elastic medium effect is depicted in Fig. (3). When the lower order frequency stands still, higher order frequency increases with enhancing stiffness. Elastic medium effectuates a gap between lower and higher order frequencies. Like the phonon gaps [59], higher order frequency ascends and increase the non-resonance area for DCNT system.

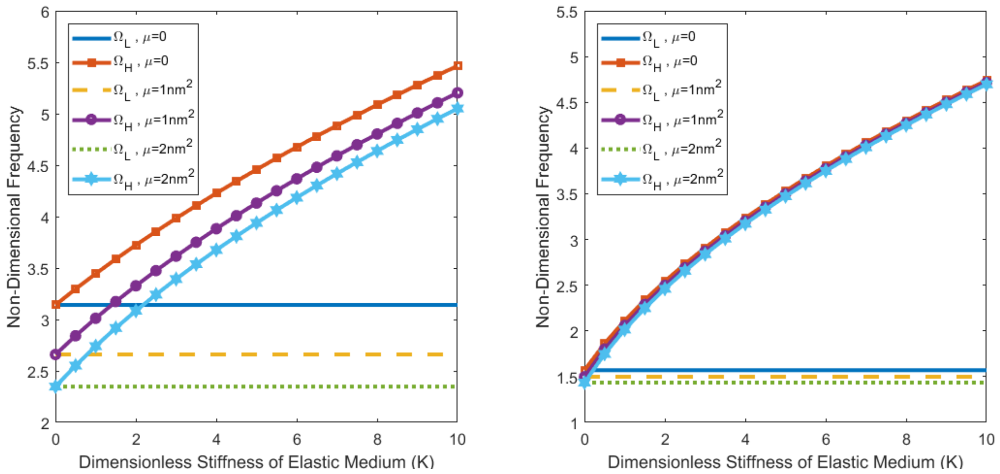

Fig. 2. Nonlocal Effect on Non-Dimensional Frequencies

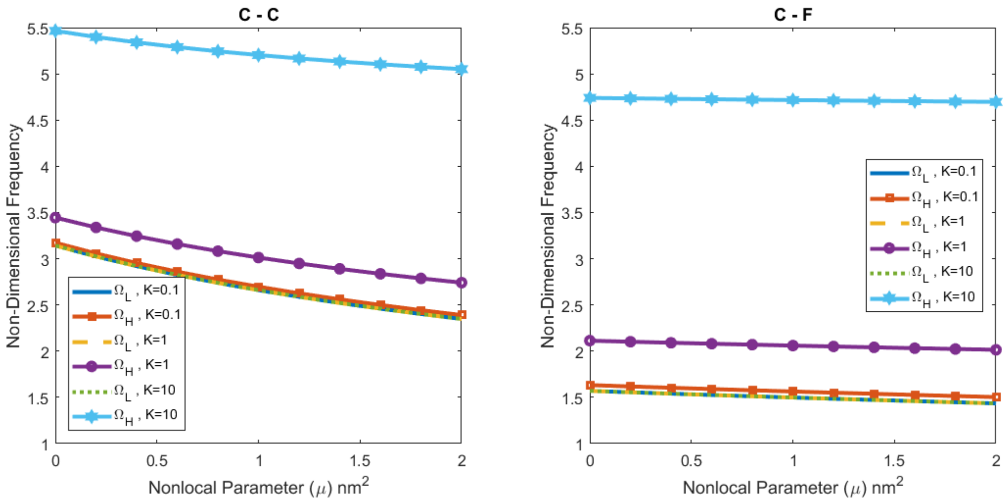

Fig. 3. Stiffness of Elastic Medium Effect Non-Dimensional Frequencies

Mode shapes for 1st mode frequencies of DCNT system is shown in Figs. (4)-(7). Both nonlocal and stiffness of elastic medium increases the amplitude of nanotubes. Stiffness has more pronounced effect on amplitude rather than nonlocality. In lower order frequency, first nanotube has negative amplitude and that means nanotubes are rotating reverse direction. This situation is called anti-phase motion. In higher order frequency, both nanotubes have positive amplitude and they are rotating same direction. This is the in-phase motion.

Amplitude ratio in Eq. (21), which depends to higher and lower order frequencies, determines whether in-phase or anti-phase motion will be occurred.

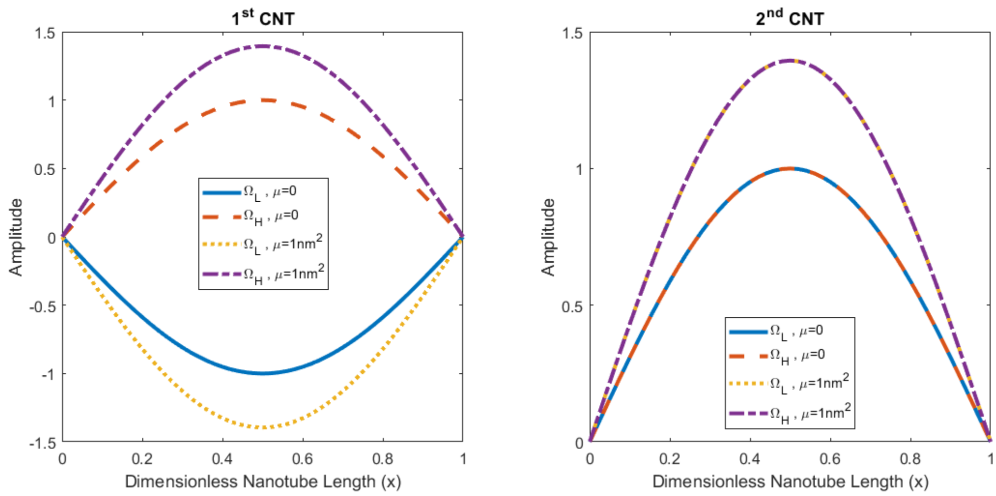

Fig. 4. Nonlocal Effect on 1st Mode Shapes of DCNT System (C-C)

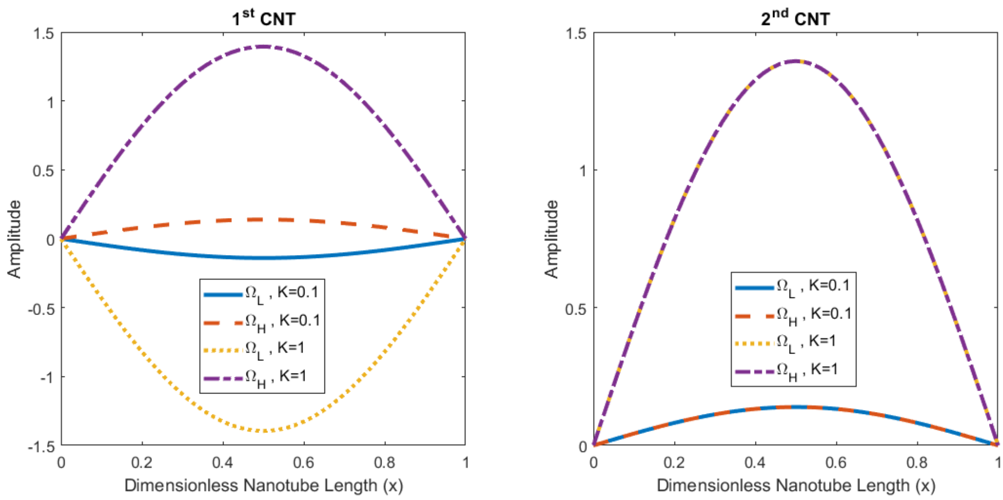

Fig. 5. Elastic Medium Effect on 1st Mode Shapes of DCNT System (C-C)

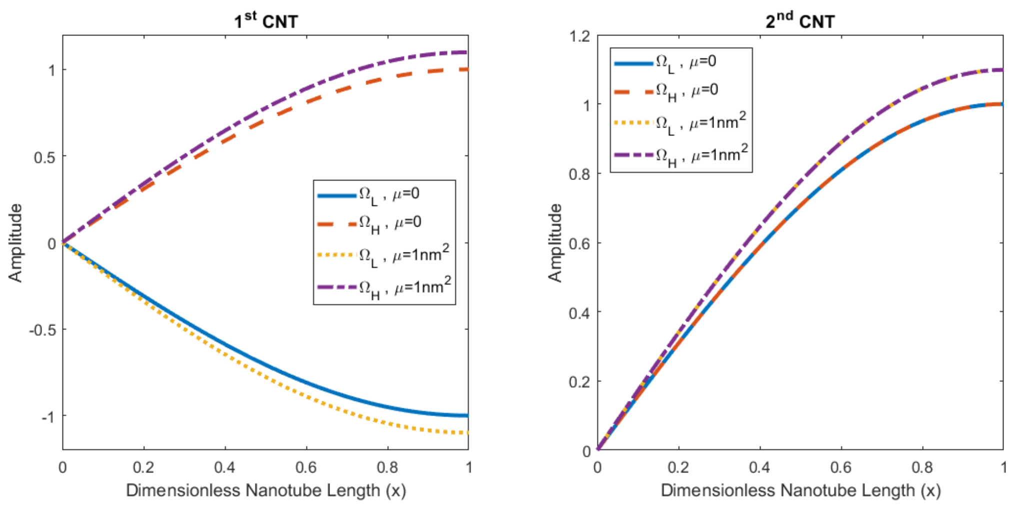

Fig. 6. Nonlocal Effect on 1st Mode Shapes of DCNT System (C-F)

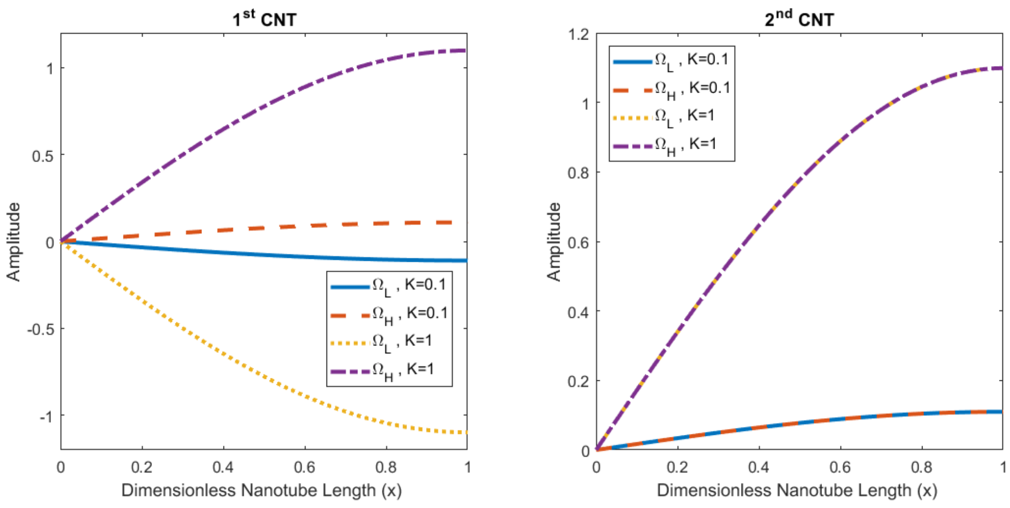

Fig. 7. Elastic Medium Effect on 1st Mode Shapes of DCNT System (C-F)

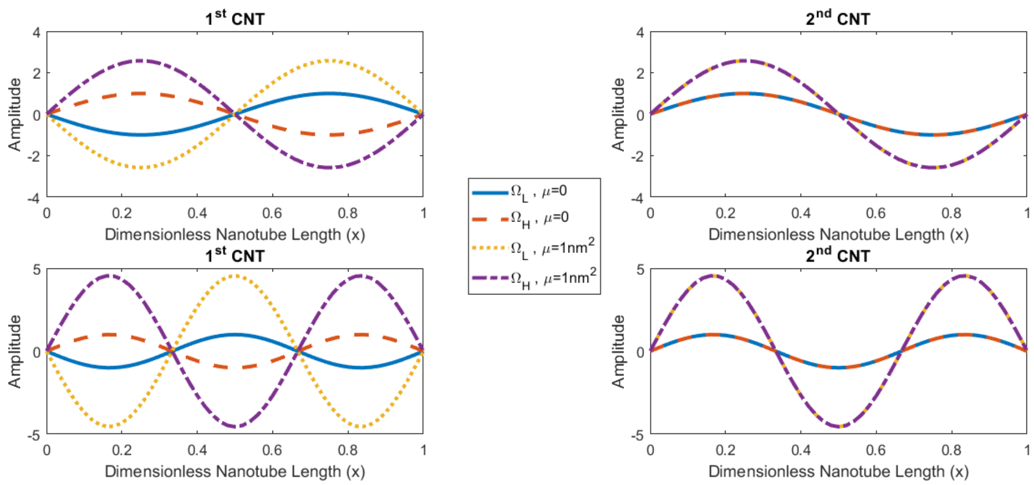

Fig. 8. Nonlocal Effect on 2nd and 3rd Mode Shapes of DCNT System (C-C)

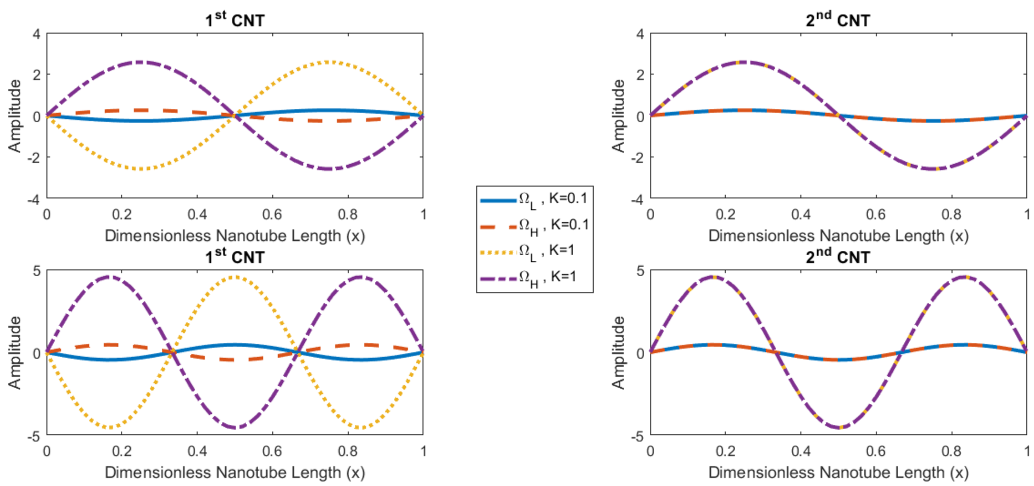

Fig. 9. Elastic Medium Effect on 2nd and 3rd Mode Shapes of DCNT System (C-C)

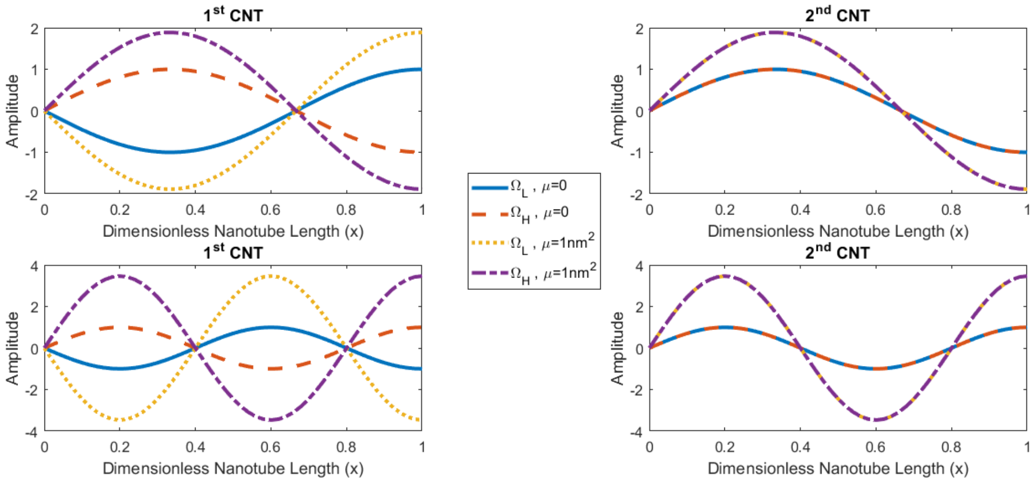

Fig. 10. Nonlocal Effect on 2nd and 3rd Mode Shapes of DCNT System (C-F)

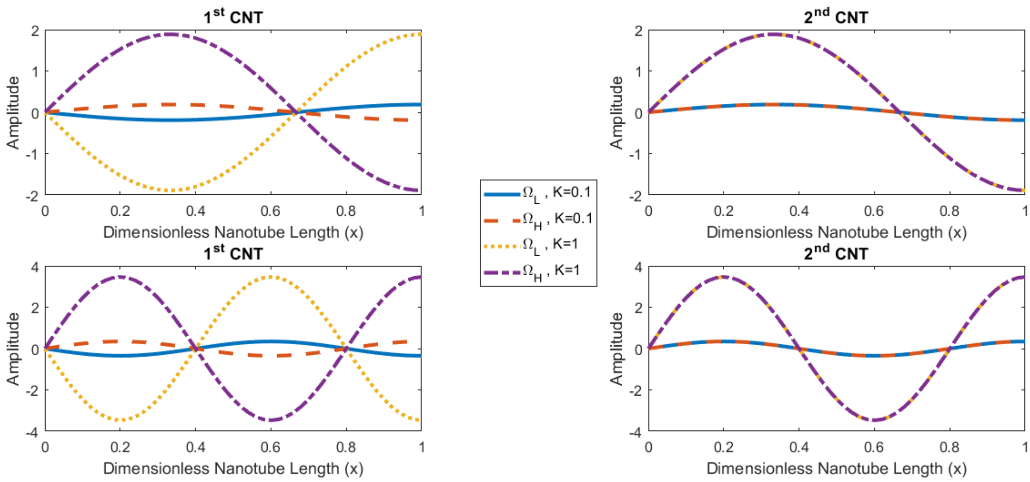

Fig. 11. Elastic Medium Effect on 2nd and 3rd Mode Shapes of DCNT System (C-F)

Conclusion

In this study, torsional vibration behavior of DCNT system is investigated. An elastic matrix is assumed between the nanotubes and it connects the nanotubes to each other. Governing equation of DCNT system is obtained with the nonlocal elasticity theory. Effects of nonlocal and stiffness of elastic medium parameters to non-dimensional frequency is studied. Following general results are obtained:

- Elastic medium creates a frequency band gap between the lower and higher order frequencies;
- Nonlocality decreases both lower and higher order frequencies with softening effect;
- Both elastic medium and nonlocality increases the amplitudes of DCNT system. Elastic medium has more pronounce effect on amplitude;
- Amplitude ratio determines the in-phase or anti-phase motion for carbon nanotubes.

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